# Séance 9 correction des exercices

## exercice 1

1. Pour $t \in [0, \alpha]$, $x_{\alpha,1}(t) = 1_{[0,\alpha]}(t) = 1 = x_\alpha(t)$ car $t \in [0,\alpha]$.

Pour $t \in ]\alpha, 1[$ $x_{\alpha,1}(t) = 1_{[0,\alpha]}(t) = 0 = x_\alpha(t)$ car $t \notin [0,\alpha]$

Donc $x_{\alpha,1}(t) = x_\alpha(t)$ pour $t \in [0, 1[$.

2. D'apres les propreétés de la série de Fourier

$$X_{\alpha,0} = \frac{1}{1}\int_0^1 x_\alpha(t)dt = \int_0^1 1_{[0,\alpha]}(t)dt.$$

$$X_{\alpha,0} = \int_0^\alpha dt = \alpha$$

3. $$H(\nu) = \int_{-\infty}^{+\infty} h(t)\, e^{-i2\pi\nu t} dt = \int_{-\infty}^{+\infty} \delta(t - 1/2)\, e^{-2i\pi\nu t} dt$$

$$H(\nu) = e^{-i\pi\nu}$$

4. D'aprèsle cours sur les filtres, on sait que la sortie est périodique quand l'entrée est périodique, et la période est la même.

Donc $y_\alpha(t)$ est périodique de période 1.

5. D'après le cours sur les filtres

$$Y_{\alpha,k} = H\left(\frac{k}{T}\right) X_{\alpha,k} \qquad \text{avec } T=1$$

Donc $Y_{\alpha,k} = e^{-i\pi k} X_{\alpha,k} = (-1)^k X_{\alpha,k}$.

6. $Y_{\alpha,0} = (-1)^0 X_{\alpha,0} = X_{\alpha,0} = \alpha.$

7. $P_{x,\alpha} = \frac{1}{1}\int_0^1 |x_{\alpha,1}|^2 dt = \int_0^1 |\mathbb{1}_{[0,\alpha]}(t)|^2 dt = \alpha$

8. $P_{x,\alpha} = \sum_{k=-\infty}^{+\infty} |X_k|^2$

$P_{y,\alpha} = \sum_{k=-\infty}^{+\infty} |Y_k|^2 = \sum_{k=-\infty}^{+\infty} |(-1)^k X_k|^2 = \sum_{k=-\infty}^{+\infty} |X_k|^2 = P_{x,\alpha}$

9. $y_{\alpha,1}(t) = \delta(t-1/2) * x_{\alpha,1}(t) = x_{\alpha,1}(t-1/2)$

$y_{\alpha,1}(t) = \mathbb{1}_{[0,\alpha]}(t-1/2) = \mathbb{1}_{[1/2,\, \alpha+1/2]}(t)$

10. D'après le cours

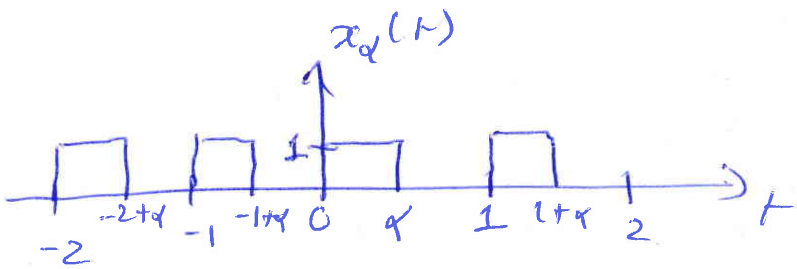
$y_\alpha(t) = \sum_{k=-\infty}^{+\infty} y_{\alpha,1}(t-k) \qquad (T=1)$

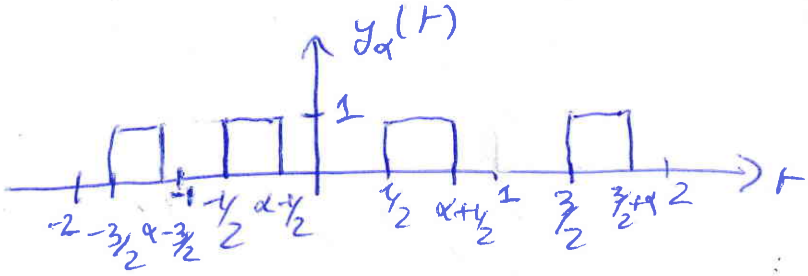
mais pour $t \notin [0,2]$, $y_{\alpha,1}(t)$ est nul

aussi si $t \in [0,1]$ $y_{\alpha,1}(t-k) = 0$ si $k \notin \{-1, 0\}$.

Donc $y_\alpha(t) = y_{\alpha,1}(t) + y_{\alpha,1}(t+1)$

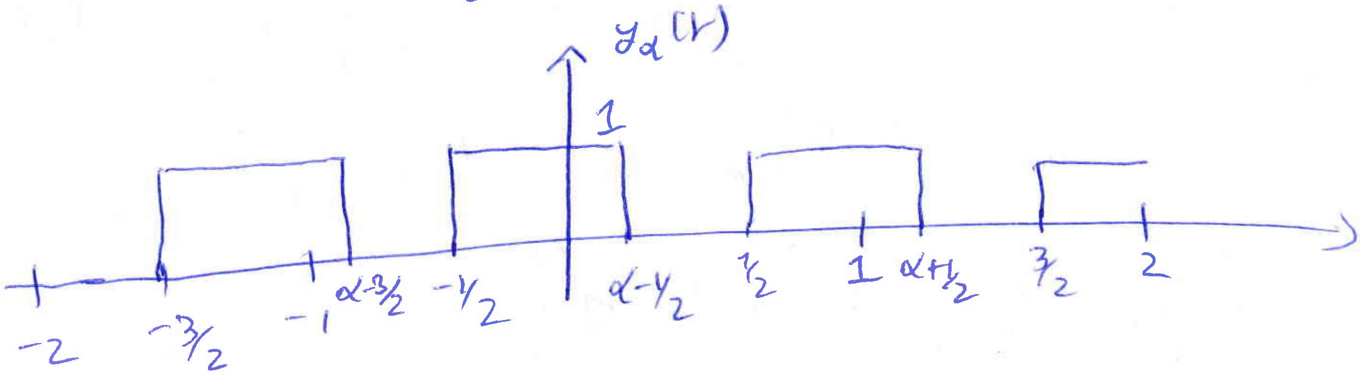
11.



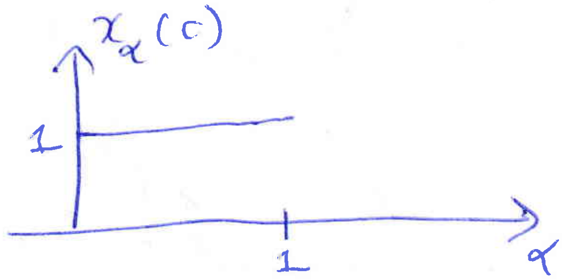



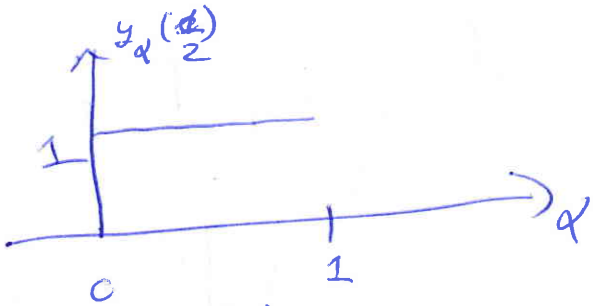

cas $\alpha \le 1/2$

cas $\alpha > 1/2$

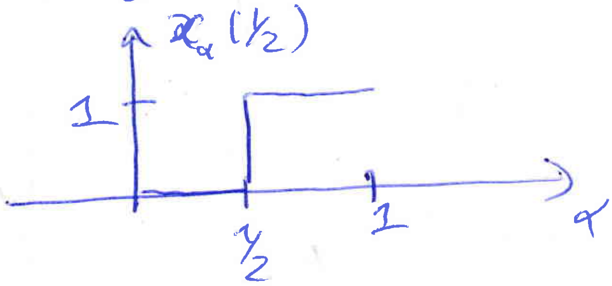


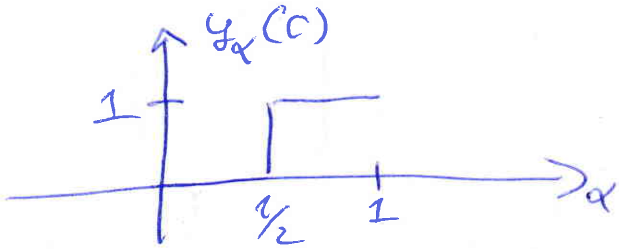

12.




13.




14. J'utilise $y_\alpha(t) = y_{\alpha,1}(t) + y_{\alpha,1}(t+1)$ pour $t \in [0,1]$

$y_\alpha(t) = \mathbb{1}_{[1/2,\, \alpha+1/2]}(t) + \mathbb{1}_{[-1/2,\, \alpha-1/2]}(t)$

Si $\alpha \leq 1/2$, $\mathbb{1}_{[-1/2,\, \alpha-1/2]}(t) = 0$

$y_\alpha(t) = \mathbb{1}_{[1/2,\, \alpha+1/2]}(t)$

Si $\alpha > 1/2$, $\quad 1\!\!1_{[-1/2,\ \alpha-1/2]}(t) = 1\!\!1_{[0,\alpha-1/2]}(t)$ pour $t \in [0,1]$

donc $\quad y_\alpha(t) = 1\!\!1_{[0,\alpha-1/2]}(t) + 1\!\!1_{[1/2,\ \alpha+1/2]}(t)$

$1\!\!1_{[1/2,\ \alpha+1/2]}(t) = 1\!\!1_{[1/2,\ 1]}(t)$

donc $\quad y_\alpha(t) = 1\!\!1_{[0,\alpha-1/2]}(t) + 1\!\!1_{[1/2,\ 1]}(t)$

15. Si $\alpha \leq 1/2$, $\quad Y_{\alpha,0} = \frac{1}{1}\int_0^1 1\!\!1_{[1/2,\ \alpha+1/2]}(t)\,dt = \int_{1/2}^{\alpha+1/2} dt = \alpha$

$$P_{y_\alpha} = \frac{1}{1}\int_0^1 \left(1\!\!1_{[1/2,\ \alpha+1/2]}(t)\right)^2 dt = \int_{1/2}^{\alpha+1/2} dt = \alpha$$

Si $\alpha > 1/2$, $\quad Y_{\alpha,0} = \frac{1}{1}\int_0^1 \left(1\!\!1_{[0,\ \alpha-1/2]}(t) + 1\!\!1_{[\frac{1}{2},\ 1]}(t)\right) dt$

$$Y_{\alpha,0} = \int_0^{\alpha-1/2} dt + \int_{\frac{1}{2}}^{1} dt = \alpha - 1/2 + 1 - 1/2$$

$$Y_{\alpha,0} = \alpha$$

$$P_{y_\alpha} = \frac{1}{1}\int_0^1 \left(1\!\!1_{[0,\ \alpha-1/2]}(t) + 1\!\!1_{[1/2,\ 1]}(t)\right)^2 dt$$

$$P_{y_\alpha} = \frac{1}{1}\left(\int_0^{\alpha-1/2} dt + \int_{1/2}^{1} dt\right) = \alpha - 1/2 + 1/2 = \alpha$$