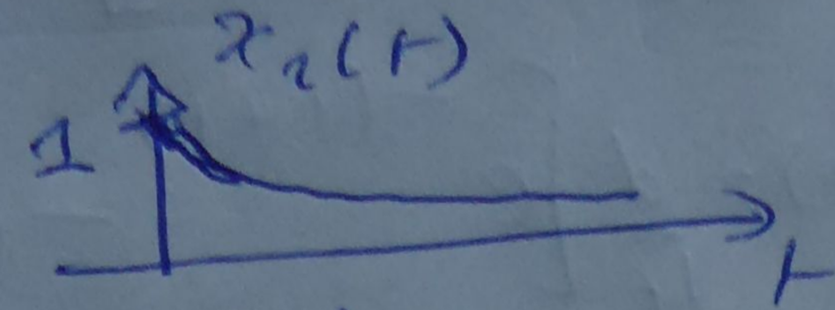
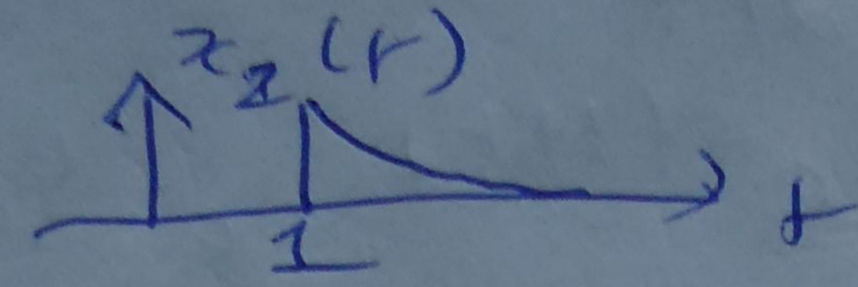
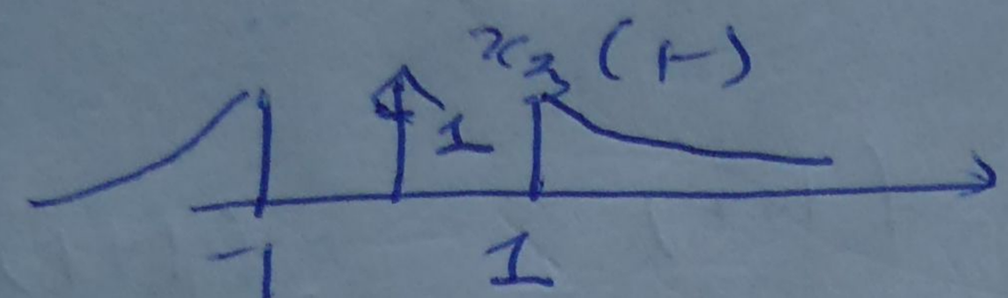
Solution exercice de séance 1

exercice 1

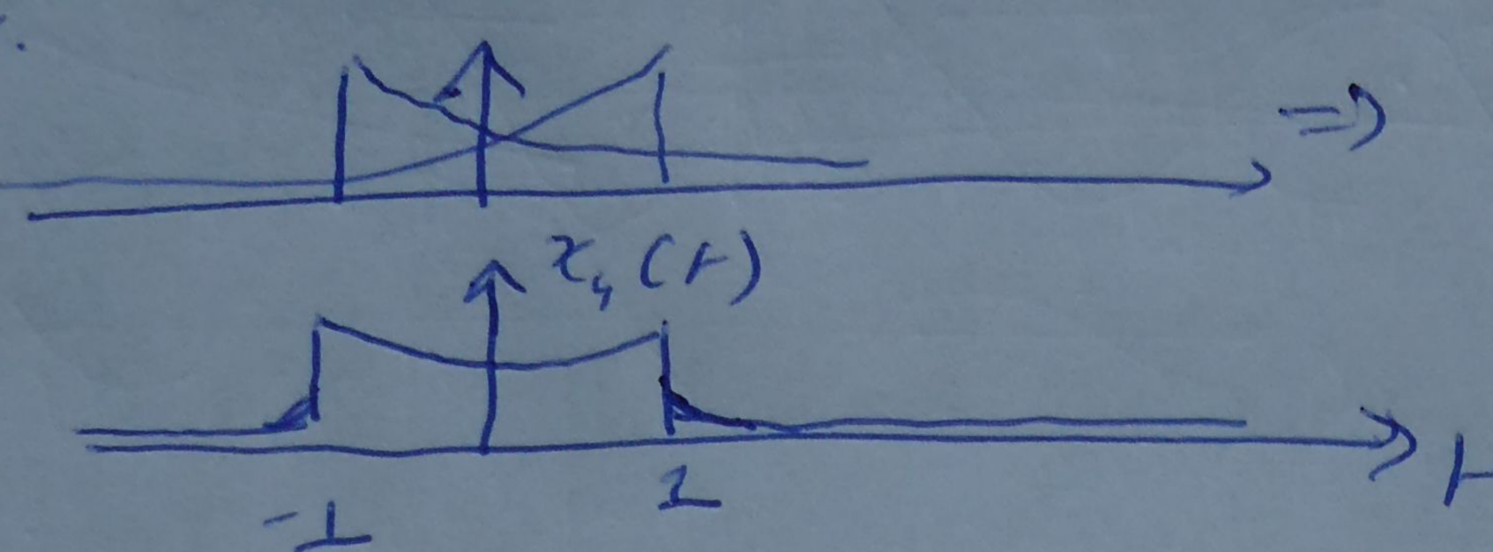
1. 

causal

2. 

3. 

$x_3(t) = x_2(t) + x_2(-t)$

4. 

pour $t<-1$ $\frac{d}{dt}x_4(t) = \frac{d}{dt}\left[e^{-(+1-t)}\mathbb{1}_{\mathbb{R}_+}(t+1)\right] > 0$

pour $t \in]-1,1[$, $\frac{d}{dt}x_4(t) = \frac{d}{dt}\left[e^{-(t+1)} + e^{-(1-t)}\right]$

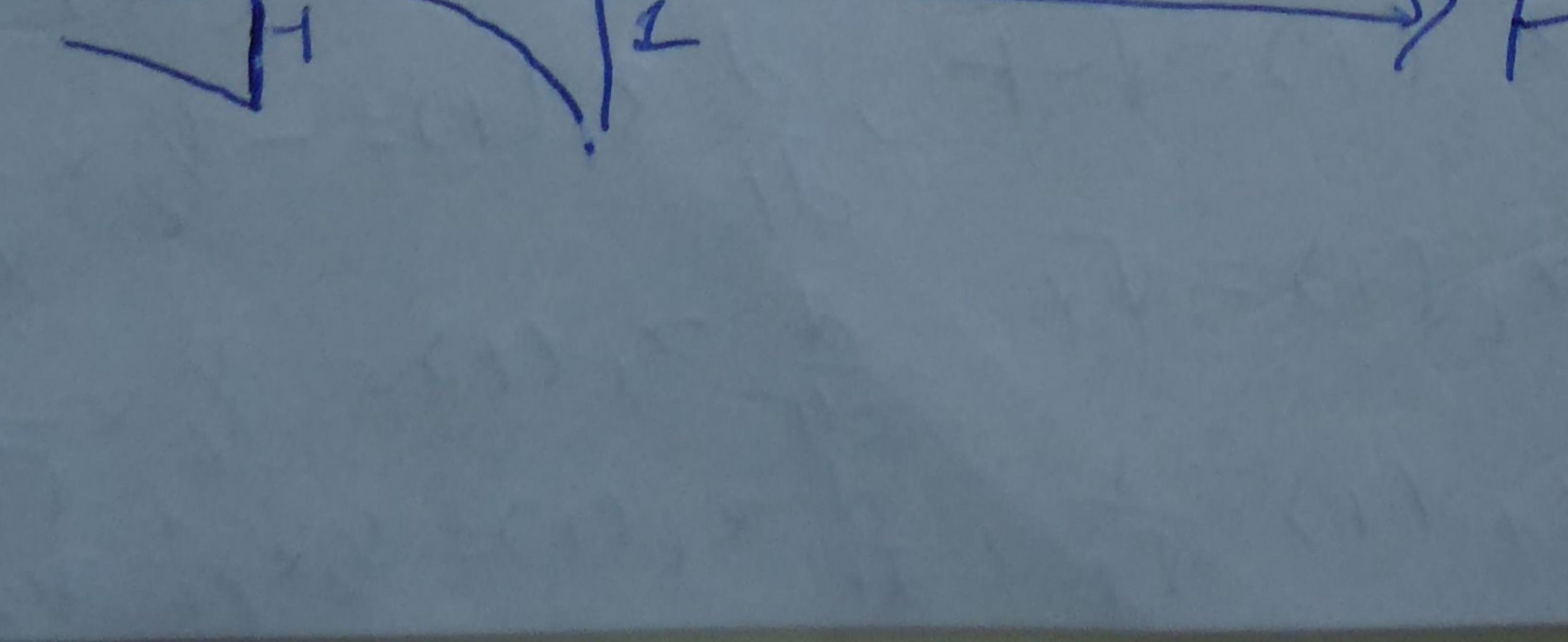
$= -e^{-(t+1)} + e^{-(1-t)}$

$= -e^{-(t+1)}\left(1 - e^{2t}\right)$

$\frac{d}{dt}x_4(t) < 0$ si $t<0$

$\frac{d}{dt}x_4(t) > 0$ si $t>0$

pour $t>1$, $\frac{d}{dt}x_4(t) = \frac{d}{dt}e^{-(1+t)} < 0$

5. 

Ex, $S_{1,2}$

pour $t<-1$, $\frac{d}{dt}x_5(t) = \frac{d}{dt}\left[-e^{-(1-t)}\right] < 0$

$t\in]-1,1]$, $\frac{d}{dt}x_5(t) = \frac{d}{dt}\left[e^{-(1+t)} - e^{-(1-t)}\right]$

$= -e^{-(1+t)} - e^{-(1-t)} < 0$

$t>1$ $\frac{d}{dt}x_5 = \frac{d}{dt}e^{-(1+t)} = -e^{-(1+t)} < 0.$

6. x_1 causal
x_2 causal
x_3 pair
x_4 pair
x_5 impair

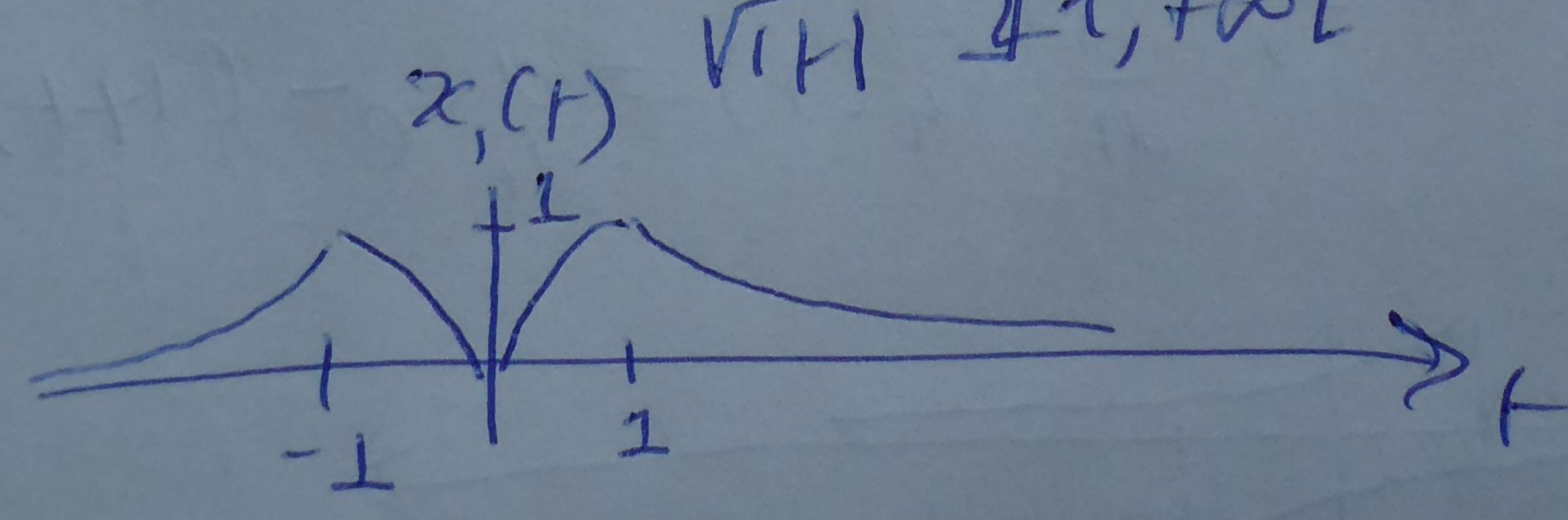
exercice 2

1. $\Pi\left(\frac{t}{2}\right) = \mathbb{1}_{[-1/2,1/2]}\left(\frac{t}{2}\right) = \mathbb{1}_{[-1,1]}(t)$

$1-\Pi\left(\frac{t}{2}\right) = 1-\mathbb{1}_{[-1,1]}(t) = \mathbb{1}_{]-\infty,-1[}(t) + \mathbb{1}_{]1,+\infty[}(t)$

$x_1(t) = \sqrt{|t|}\,\mathbb{1}_{[-1,1]}(t) + \frac{1}{\sqrt{|t|}}\mathbb{1}_{]-\infty,-1[}(t)$

$+ \frac{1}{\sqrt{|t|}}\mathbb{1}_{]1,+\infty[}(t)$



2. $t<-1$, $x_1(t) = \frac{1}{\sqrt{-t}}$, $\frac{d}{dt}x_1(t) = \frac{+1/2}{(-t)^{3/2}}$

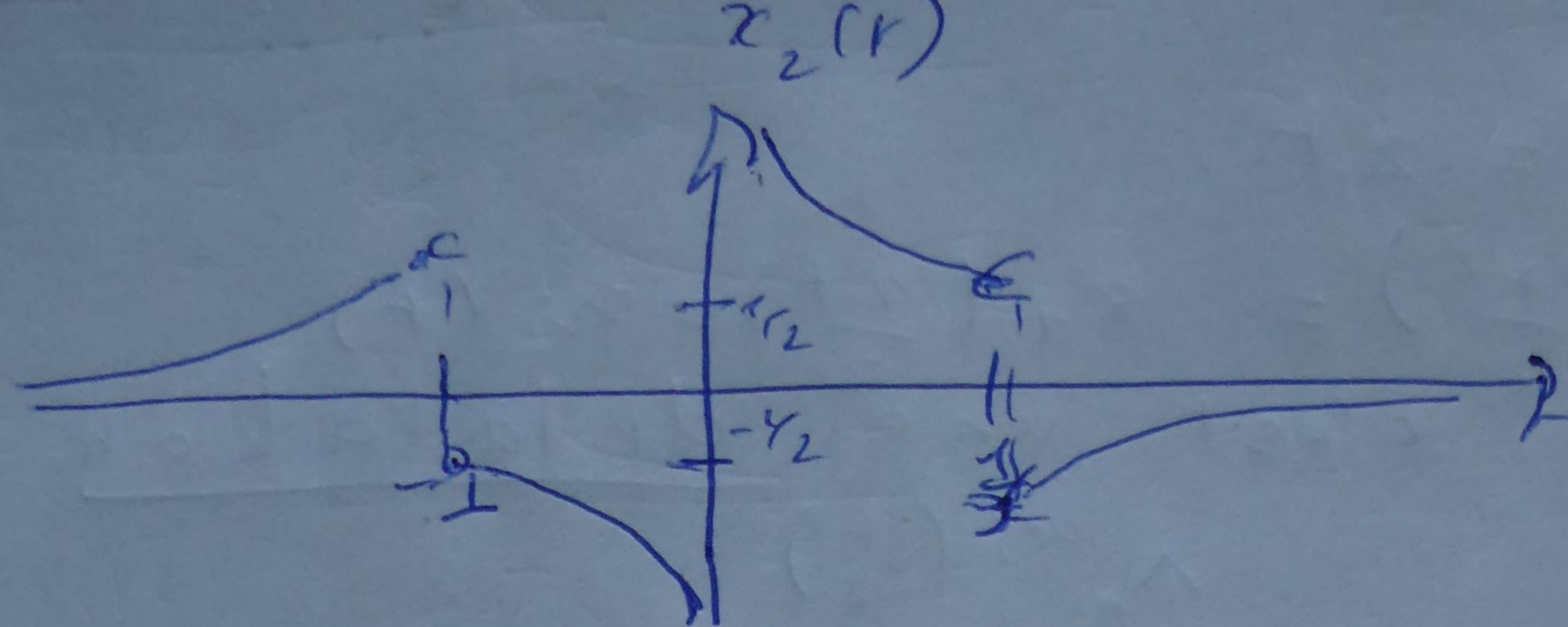
$-1<t<0$, $x_1(t) = \sqrt{-t}$, $\frac{d}{dt}x_1(t) = -\frac{1}{2}\times\frac{1}{\sqrt{-t}}$

$0<t<1$, $x_1(t) = \sqrt{t}$, $\frac{d}{dt}x_1(t) = \frac{1}{2}\times\frac{1}{\sqrt{t}}$

$t>1$, $x_1(t) = \frac{1}{\sqrt{t}}$, $\frac{d}{dt}x_1(t) = \frac{1}{2}\times\frac{1}{(t)^{3/2}}$

Ex S2,3

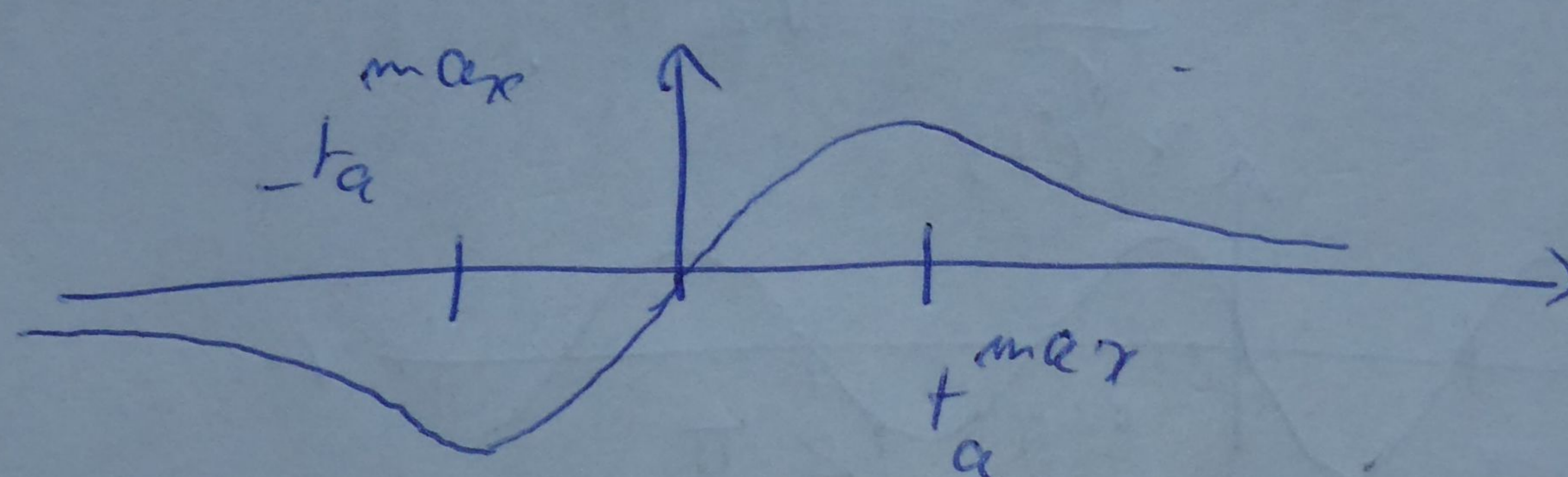
$$x_2(t) = \frac{1}{2}\,\mathrm{sign}(t)\,\Pi\left(\frac{t}{2}\right)\frac{1}{\sqrt{|t|}} - \frac{1}{2}\,\mathrm{sign}(t)\left(1-\Pi\left(\frac{t}{2}\right)\right) \times \frac{1}{|t|^{3/2}}$$



exercice 3

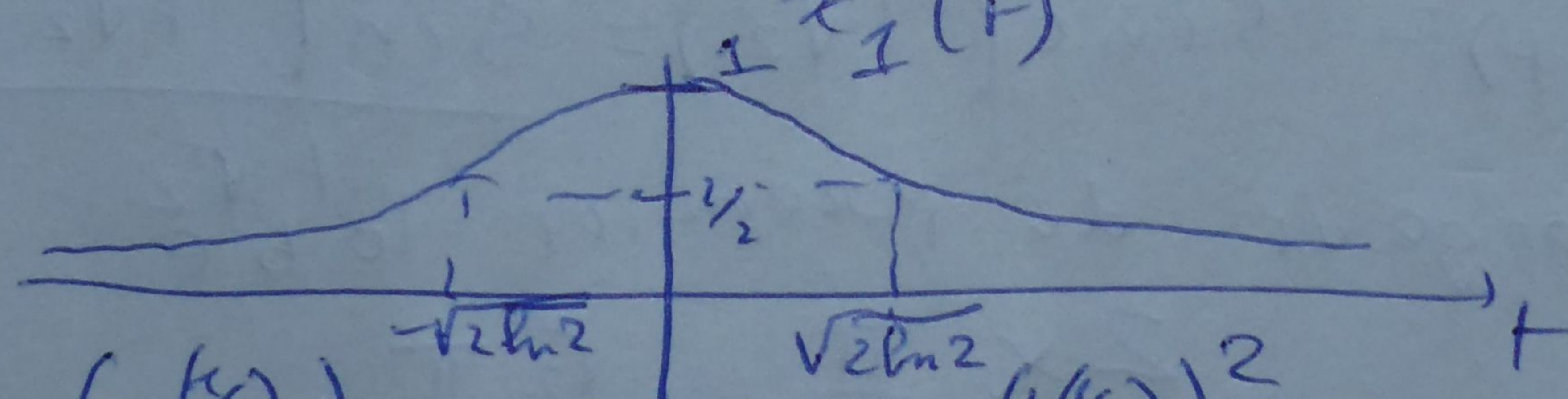
1. $$\frac{d}{dt}x_a(t) = e^{-at^2} + t(-2at)\,e^{-at^2}$$
$$= e^{-at^2}\left(1-2at^2\right)$$

$$t_a^{max} = \frac{1}{\sqrt{2a}}$$



x_a impair

2.



$$x_1\left(t^{(1/2)}\right) = \frac{1}{2} \iff e^{-\frac{\left(t^{(1/2)}\right)^2}{2}} = \frac{1}{2}$$

$$\iff -\frac{\left(t^{(1/2)}\right)^2}{2} = -\ln 2$$

$$\iff t^{(1/2)} = \sqrt{2\ln 2} \qquad \left(t^{(1/2)} > 0\right).$$

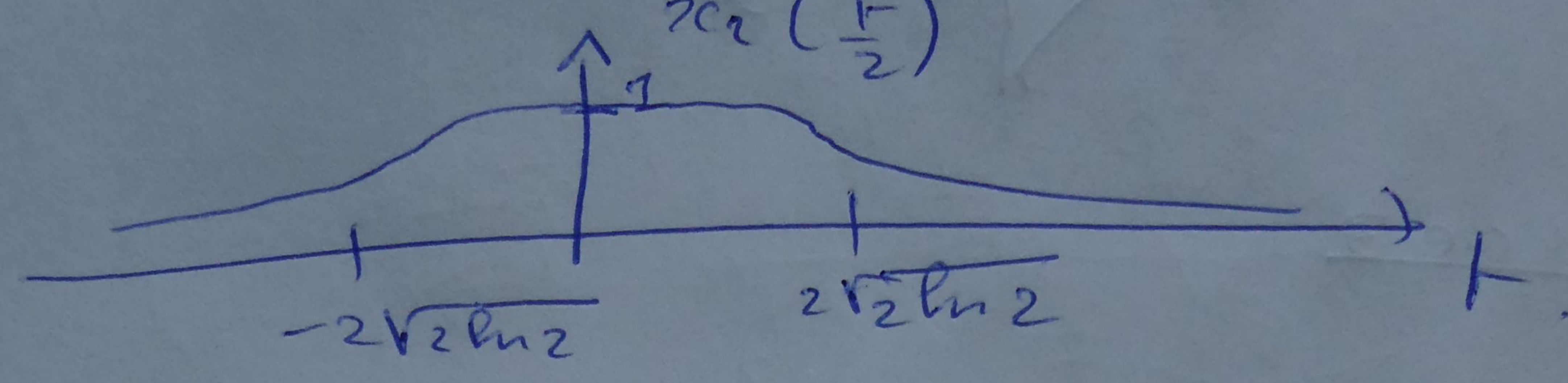
Ex S_1, 4

3. $x_1\left(\frac{2t^{(1/2)}}{2}\right) = x_1\left(t^{(1/2)}\right) = \frac{1}{2}$.

$t' = 2t$

$x_1\left(\frac{t'}{2}\right) = x_1\left(\frac{2t}{2}\right) = x_1(t)$

Donc c'est une <u>dilatation</u>

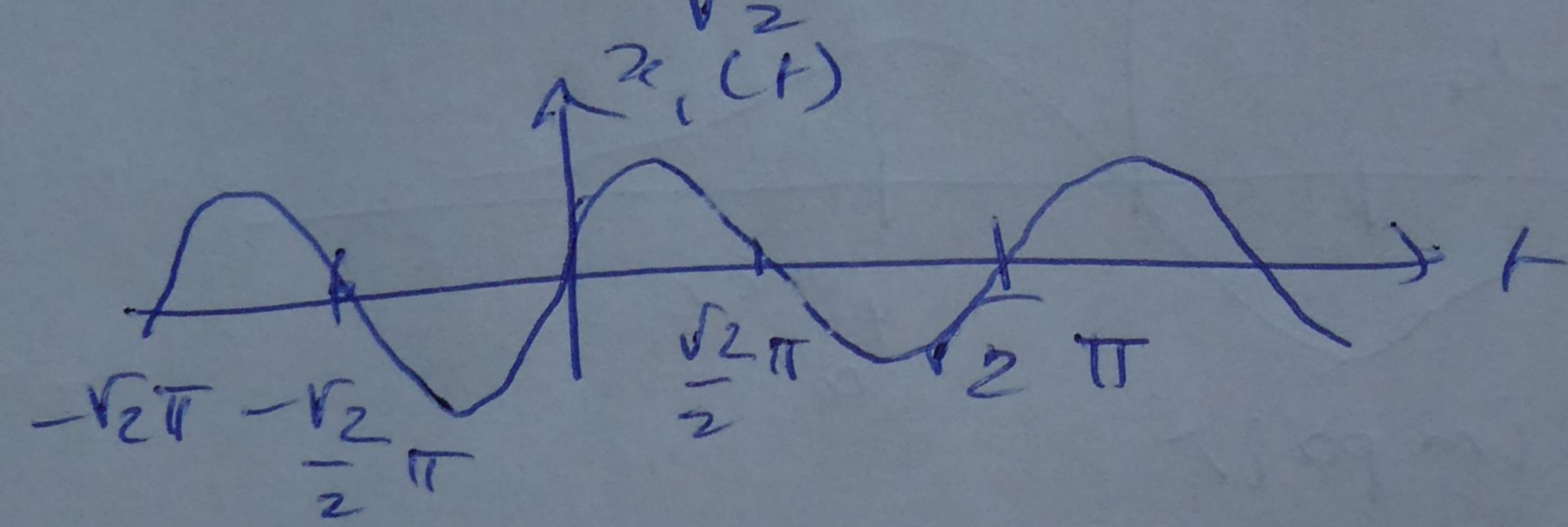


<u>exercice 4</u>

1. $x_1(t) = \sin(t\sqrt{2}) = \sin\left(2\pi \frac{t\sqrt{2}}{2\pi}\right)$

périodique de période

$$T = \frac{2\pi}{\sqrt{2}} = \sqrt{2}\pi.$$

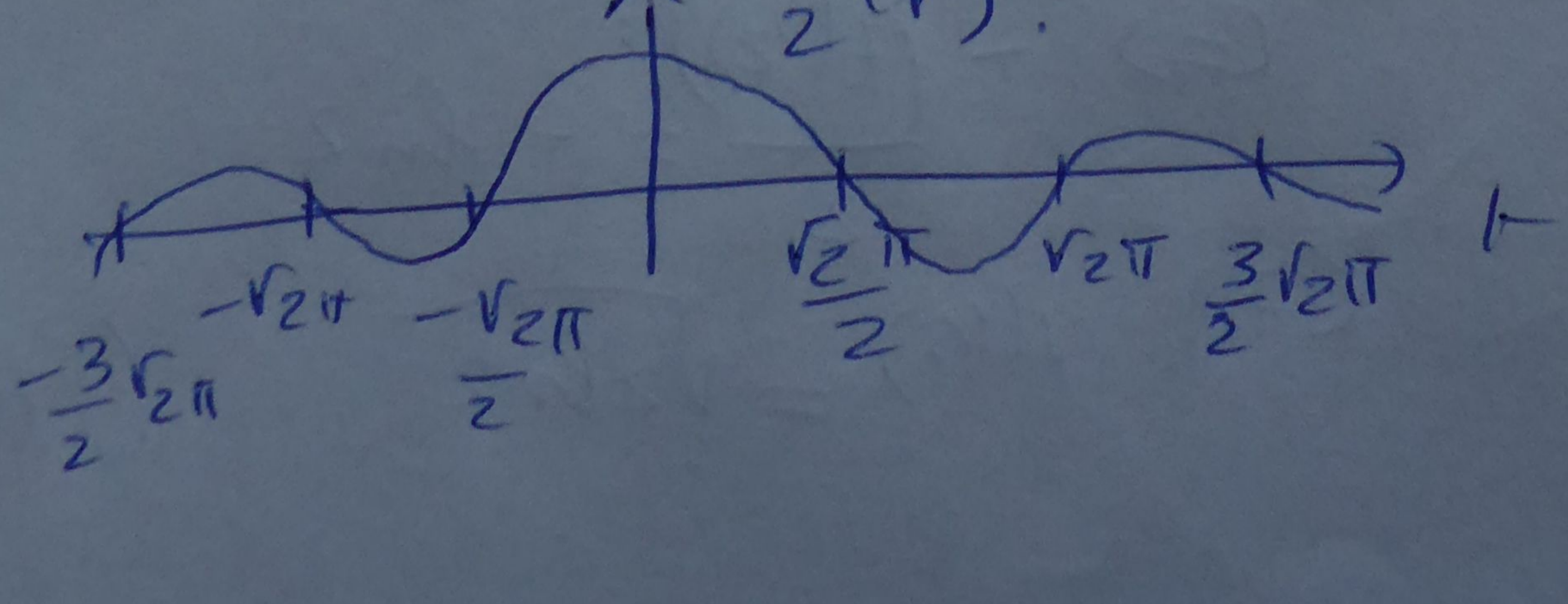


2. $x_2(t) = \mathrm{sinc}(t\sqrt{2}) = \frac{\sin(t\sqrt{2})}{t\sqrt{2}}$

Largeur du premier lobe

$$2 \times \frac{\pi}{\sqrt{2}} = \sqrt{2}\pi.$$

Largeur des autres lobes : $\frac{\sqrt{2}\pi}{2}$



Ex S1,5

3. $x_3(t) = \sin^2(t\sqrt{2})$

$$= 1 - \cos^2(t\sqrt{2})$$

$$= 1 - \left(\frac{1}{2} + \frac{1}{2}\cos(2t\sqrt{2})\right)$$

$$= \frac{1}{2} - \frac{1}{2}\cos(2t\sqrt{2})$$

C'est périodique de période $\frac{\pi}{\sqrt{2}} = \frac{\sqrt{2}\,\pi}{2}$,

$\cos(2t\sqrt{2}) = \cos\left(2\pi\, t\left(\frac{\sqrt{2}}{\pi}\right)\right)$