Question 1

F. A. $\int_{-\infty}^{+\infty} \delta(t+1)\, x(t)\, dt = x(-1) = (-1)^2 + 3(-1) + 2$
$= 1 - 3 + 2 = 0$

F. B. $\int_{-\infty}^{+\infty} \delta'(t+1)\, x(t)\, dt = -x'(-1) = -1$

$x'(t) = 2t + 3$

$x'(-1) = -2 + 3 = 1$

F. C. $\int_{-\infty}^{+\infty} vp\left(\frac{1}{t+1}\right) x(t)\, dt = \int_{-\infty}^{+\infty} (t+2) \mathbb{1}_{[-2,2]}(t)\, dt$

$x(t) = (t+1)(t+2) \mathbb{1}_{[-2,2]}(t)$

$= \int_{-2}^{2} (t+2)\, dt = \left[t^2 + 2t \right]_{-2}^{2} = 4 + 4 - (4 - 4)$

$= 8$

V. D. $\int_{-\infty}^{+\infty} H(t)\, x(t)\, dt = \int_{-2}^{2} H(t)\, (t^2 + 3t + 2)\, dt$

$= \int_0^2 (t^2 + 3t + 2)\, dt = \left[\frac{t^3}{3} + \frac{3t^2}{2} + 2t \right]_0^2$

$= \frac{8}{3} + \frac{3 \times 4}{2} + 4 = 3\frac{-1}{3} + 6 + 4 = 13\frac{-1}{3}$

Question 2

F A. $x(t)$ est périodique non-nul
donc $E_x = +\infty$.

V B. $x(t)$ est périodique de période 2π

Sur 1 période $\begin{cases} x(t) = 1 & \text{pour } t \in [0, \pi/3] \text{ et } t \in [2\pi - \pi/3, 2\pi] \\ x(t) = 0 & \text{pour } t \in]\pi/3, 2\pi - \pi/3[\end{cases}$

$$P_x = \frac{1}{2\pi} \times \left[1 \times \pi/3 + 1 \times \pi/3\right] = (2\pi/3) \times \frac{1}{2\pi}$$

$$P_x = 1/3.$$

V C. $$E_y = \int_{-\infty}^{+\infty} \left(e^{-|t|}\right)^2 dt = \underbrace{2\int_0^{+\infty} e^{-2t}\, dt}_{y(t) \text{ est pair}}$$

$$E_y = 2\left[-\frac{e^{-2t}}{2}\right]_0^{+\infty} = 2\left(\frac{1}{2}\right) = 1.$$

F. D. $y(t)$ non périodique avec $E_y < +\infty$
donc $P_y = 0$.

Question 3

F. A. $$Y_0 = \frac{1}{1}\int_0^1 y(t)\,dt = \int_0^1 x(t)\,dt = \frac{1}{2}$$

$$Z_0 = \frac{1}{2}\int_0^2 y(t)\,dt = \frac{1}{2}\int_0^1 x(t)\,dt = \frac{1}{4}.$$

F. B. Comme $y(t)$ est périodique de période 1, $y(t-1) = y(t)$ et $z(t) \neq 2y(t)$.
Par contre $y(t) = z(t) + z(t-1)$

V. C. $$P_y = \frac{1}{1}\int_0^1 |y(t)|^2 dt = \frac{1}{1}\int_0^1 x^2(t)\,dt = E_x$$

V D. $z_k = \frac{1}{2}\int_0^2 y(t)\, e^{-2i\pi \frac{k}{2} t}\, dt = \frac{1}{2}\int_0^1 x(t)\, e^{-i\pi k t}\, dt$

$$\frac{1}{2} X\left(\frac{k}{2}\right) = \frac{1}{2}\int_{-\infty}^{+\infty} x(t)\, e^{-2i\pi \frac{k}{2} t}\, dt = z_k$$

Question 4

V A. $E_x = \int_{-\infty}^{+\infty} |x(t)|^2\, dt = \int_{-1/2}^{1/2} \left|e^{i2\pi t}\right|^2 dt = \int_{-1}^{1/2} dt = 1$

V B. $X(1) = \int_{-\infty}^{+\infty} x(t)\, e^{-2i\pi t}\, dt = \int_{-1/2}^{1/2} e^{i2\pi t}\, e^{-2i\pi t}\, dt$

$$= \int_{-1/2}^{1/2} dt = 1.$$

V.C. $\left(x(t) * x(t)\right)(0) = \int_{-\infty}^{+\infty} x(\tau)\, x(-\tau)\, d\tau$

$$= \int_{-1/2}^{1/2} e^{2i\pi t}\, e^{-2i\pi t}\, dt = 1.$$

V.D. $\varphi_{xx}(0) = \int_{-\infty}^{+\infty} x(\tau)\, x(\tau)^*\, d\tau$

$$= \int_{-1/2}^{1/2} e^{2i\pi\tau}\, e^{-2i\pi\tau}\, d\tau = \int_{-1/2}^{1/2} d\tau = 1$$

Question 5

V A. $P_x = \frac{1}{2}\left(\int_0^2 |x(t)|^2 dt\right)$

$P_x = \frac{1}{2}\left(\int_0^1 |1|^2 dt + \int_1^2 |i|^2 dt\right)$

$P_x = \frac{1}{2}(1 + 1) = 1$.

F. B. $X_1 = \frac{1}{2}\int_0^2 x(t) e^{-i2\pi\frac{1}{2}t} dt$

$X_1 = \frac{1}{2}\int_0^1 e^{-i\pi t} dt + \frac{1}{2}\int_1^2 i e^{-i\pi t} dt$

$Re(X_1) = \frac{1}{2}\int_0^1 \cos \pi t \, dt + \frac{1}{2}\int_1^2 \sin \pi t \, dt$

$Re(X_1) = \frac{1}{2}\left[\frac{\sin \pi t}{\pi}\right]_0^1 + \frac{1}{2}\left[\frac{-\cos \pi t}{\pi}\right]_1^2$

$Re(X_1) = \frac{1}{2}\left(\frac{-1}{\pi} - \frac{1}{\pi}\right) = \frac{-1}{\pi}$

V. C. $X_0 = \frac{1}{2}\int_0^2 x(t) dt = \frac{1}{2}\int_0^1 dt + \frac{1}{2}\int_1^2 i dt$

$X_0 = \frac{1}{2} + \frac{1}{2} i = \frac{1}{2}(1+i)$

$|X_0| = \frac{1}{2}|1+i| = \frac{\sqrt{2}}{2}$

F. D. $x(t-1) = i x(t)$ est vrai pour $t \in [1, 1[$.
mais $1 = x(\frac{1}{2}) \neq i x(\frac{3}{2}) = -1$