Qr 19,1

Séance 10

Question 1

V. A. Soit $y(t) = x(t+a)$ avec $a>0$
$y(t)$ est en avance par rapport à $x(t)$.

$y(t) * y(t) = (x(t) * x(t))(t+2a)$

En effet
$$y(t) * y(t) = \int_{-\infty}^{+\infty} y(\tau)\, y(t-\tau)\, d\tau = \int_{-\infty}^{+\infty} x(\tau+a)\, x(t-\tau+a)\, d\tau$$
$$= \int_{-\infty}^{+\infty} x(\tau')\, x(t-\tau'+2a)\, d\tau'$$
$\tau' = \tau + a$: changement de variable
$$= (x(t) * x(t))(t+2a).$$

F. B. C'est faux. $\varphi_{xx}(t)$ est toujours pair indépendamment de ce que l'on retarde ou avance $x(t)$.
$$\varphi_{yy}(t) = \int_{-\infty}^{+\infty} y(\tau)\, y^*(\tau-t)\, d\tau = \int_{-\infty}^{+\infty} x(\tau+a)\, x^*(\tau+a-t)\, d\tau$$
$$= \int_{-\infty}^{+\infty} x(\tau)\, x^*(\tau-t)\, d\tau = \varphi_{xx}(t).$$

V. C. $$(x(t) * x(t))(-t) = \int_{-\infty}^{+\infty} x(\tau)\, x(-t-\tau)\, d\tau$$
$$= \int_{-\infty}^{+\infty} x(-\tau)\, x(t+\tau)\, d\tau$$
changement de variable $\tau' = -\tau$
$$= \int_{-\infty}^{+\infty} x(\tau)\, x(t-\tau)\, d\tau$$
$$= (x(t) * x(t))(t).$$

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✓ D. Indépendamment de $x(t)$, $\varphi_{xx}(\tau)$ est toujours pair tant que $x(t)$ est pair.

$$\varphi_{xx}(-t) = \int_{-\infty}^{+\infty} x(\tau)\, x(\tau+t)^* d\tau$$

$$= \int_{-\infty}^{+\infty} x(-\tau)\, x(-\tau-t)^* d\tau$$

$\tau' = -\tau - t$ alors $\tau = -\tau' + t$

$$\varphi_{xx}(-t) = \int_{-\infty}^{+\infty} x(\tau'-t)\, x(\tau')^* d\tau.$$

$$= \varphi_{xx}(t)^*.$$

Question 2

V A. $y_2(t) = h(t) * x_2(t) = h(t) * (x_1(t) + x_1(t-1))$

$= h(t) * x_1(t) + (h(t) * x_1(t-1))$

$= y_1(t) + y_1(t-1)$

F. B. $\varphi_{x_2x_2}$ est pair, alors que $\varphi_{x_1x_1}(t) + \varphi_{x_1x_1}(t-1)$ ne l'est pas.

V. C. $(h(t) + h(t-1)) * x_1(t)$

$= h(t) * x_1(t) + h(t-1) * x_1(t)$

$= y_1(t) + h(t) * x_1(t-1)$

$= h(t) * (x_1(t) + x_1(t-1))$

$= h(t) * x_2(t) = y_2(t).$

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✓ D. $\varphi_{x_2x_2}(t) = x_2(t) * x_2(-t)$

$= \left(x_1(t) + x_1(t-1)\right) * \left(x_1(-t) + x_1(-t-1)\right)$

$= x_1(t) * x_1(-t) + x_1(t) * x_1(-t-1)$

$+ x_1(t-1) * x_1(-t) + x_1(t-1) * x(-t-1)$

$= \varphi_{x_1x_1}(t) + \left(x_1(+t) * x_1(-t)\right)(t+1)$

$+ \left(x_1(t) * x_1(-t)\right)(t-1) + x_1(t) * x(-t)$

$= \varphi_{x_1x_1}(t) + \varphi_{x_1x_1}(t+1) + \varphi_{x_1x_1}(t-1)$

$+ \varphi_{x_1x_1}(t).$

Question 4

✓ A. $x_1(t) = 1\!\!1_{[-1/2,\,1/2]}(t - 1/2)$

$x_1(t) * x_1(t) = \left(1\!\!1_{[-1/2,\,1/2]}(t) * 1\!\!1_{[-1/2,\,1/2]}(t)\right)(t-1)$

$\varphi_{x_1x_1}(t) = \varphi_{x_0x_0}(t)$

$x_0(t) = 1\!\!1_{[-1/2,\,1/2]}(t)$

F. B. $x_2(t) = x_1\left(\frac{t}{2}\right)$

solution 1 → $X_2(\nu) = 2X_1(2\nu)$

$|X_2(\nu)|^2 = 4|X_1(2\nu)|^2$

$TF[\varphi_{x_2x_2}(t)] = 4|X_1(2\nu)|^2$

$\varphi_{x_2x_2}(t) = \frac{4}{2}\varphi_{x_1x_1}\left(\frac{t}{2}\right) = 2\left(1 - \left|\frac{t}{2}\right|\right)1\!\!1_{[-2,2]}(t)$

solution 2

→ $\left(x\left(\frac{t}{2}\right) * y\left(\frac{t}{2}\right)\right)(t) = \int_{-\infty}^{+\infty} x\left(\frac{\tau}{2}\right) y\left(\frac{t}{2} - \frac{\tau}{2}\right) d\tau$

$\tau' = \frac{\tau}{2} \quad d\tau' = \frac{d\tau}{2} \quad = 2\int_{-\infty}^{+\infty} x(\tau')\, y\left(\frac{t}{2} - \tau'\right) d\tau'$

$= 2\left(x(t) * y(t)\right)\left(\frac{t}{2}\right).$

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V. C. Certes $1\!\!1_{[-2,0]}(t)$ et $1\!\!1_{[0,2]}(t)$ ne sont ni pairs, ni impairs.

Mais $1\!\!1_{[-2,0]}(t) = 1\!\!1_{[0,2]}(-t)$

$$\left(1\!\!1_{[-2,0]}(t) * 1\!\!1_{[0,2]}(t)\right)(t)$$

$$= 1\!\!1_{[0,2]}(-t) * 1\!\!1_{[0,2]}(t) = \varphi_{x_3 x_3}(t)$$

$$x_3(t) = 1\!\!1_{[0,2]}(t).$$

$\varphi_{x_3 x_3}(t)$ est pair.

V D. $1\!\!1_{[0,2]}(t) = 1\!\!1_{[-2,0]}(t-2)$

$$1\!\!1_{[0,2]}(t) * 1\!\!1_{[0,2]}(t) = \left(1\!\!1_{[0,2]}(t) * 1\!\!1_{[-2,0]}(t)\right)(t-2)$$

Question 3

$$x(t) = 1\!\!1_{]-\infty,1]}(t)$$

F A. $x(t-1) = 1\!\!1_{]-\infty,2]}(t)$

V B.

V C. $x(-t) = 1\!\!1_{[-1,+\infty[}(t)$

$$x(-(t-1)) = 1\!\!1_{[0,+\infty[}(t)$$

$$x(1-t) = 1\!\!1_{[0,+\infty[}(t)$$

V D. $x(-t) = 1\!\!1_{[-1,+\infty[}(t)$

$$x(t)x(-t) = 1\!\!1_{[-1,1]}(t)$$

Question 5

F. A. $\varphi_{xx}(t) = \int_{-\infty}^{+\infty} x(\tau)\, x(\tau-t)^* \, d\tau$ pour $t \in [0,1]$

$\varphi_{xx}(t) = \int_{0}^{1} x(\tau)\, x(\tau-t)^* \, d\tau$ car $x(\tau)=0$ si $\tau \notin [0,1]$.

$\varphi_{xx}(t) = \int_{t}^{1} x(\tau)\, x(\tau-t)^* \, d\tau$ car $x(\tau-t)=0$ si $\tau \in [0,t]$.

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F B. $\varphi_{xx}(-t) = \varphi_{xx}(t)^* \neq \varphi_{xx}(t)$

V C. $\varphi_{xx}(0) = \int_{-\infty}^{+\infty} x(\tau)\, x(+\tau)^* d\tau$

$$= \int_0^1 |x(\tau)|^2 d\tau = \underline{1}$$

V D. $\varphi_{xx}(1) = \int_{-\infty}^{+\infty} x(\tau)\, x(\tau-1)^* d\tau$

$$= \int_0^1 x(\tau)\, x(\tau-1)^* d\tau = 0$$

car $x(\tau-1) = 0$ si $\tau \in [0, 1]$.