EXTENDED DISPARITY MAP ESTIMATION ALGORITHM USING JOINT ENTROPY-DISTORTION METRIC FOR NON-RECTIFIED STEREOSCOPIC IMAGES

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ABSTRACT
This paper deals with the block-based matching problem in a stereoscopic image to estimate the disparity map. In most cases, matches are chosen according to the minimum mean square error criterion. However, for matching a block, several disparities may be potential candidates as they meet the minimum distortion. Unfortunately this latter may not be consistent with the reduction of the stereoscopic image encoding cost. To address this problem, an optimization algorithm using an entropy-distortion metric is proposed. The selected disparities reduce not only the distortion of the predicted image but also the entropy of the estimated disparity map under a low computational complexity. The developed algorithm is based on the underlying idea of the generic $M$-algorithm where many changes were required and have been made to fit our problematic. Simulation results on stereoscopic images show that our optimization algorithm achieves better rate-distortion performance than the traditional block-matching algorithm.

Index Terms— Stereoscopic image, block matching, optimization, entropy, disparity

1. INTRODUCTION
The success of 3D-movies, the growing number of 3D-displays within households and the increasing amount of 3D-content lead to a strong demand for a fast development of efficient stereoscopic image and video coding solutions. A stereoscopic image is composed of two views; the left and right views which are captured from the same scene with slightly different viewpoints sharing therefore a large amount of redundancies. As a result, objects from the left view can be seen on the right view with a little shift called disparity. Most stereoscopic image coding algorithms provided in the state-of-the-art rely on the following main steps: (i) coding the selected reference view, for example the left view; (ii) estimating and encoding the disparity map; (iii) predicting the right view using the decoded left view and the estimated disparity map; and (iv) coding the difference between the predicted right view and the decoded left view. Although the bit-rate required to encode the disparity map should not be overlooked, this stereoscopic encoding strategy is proven to be better than encoding each view separately. The proposed paper addresses the problem of estimating the disparity map that achieves the best view prediction subjected to a given bit-rate to encode the estimated disparity map.

Most state-of-the-art methods dealing with the estimation of the disparity map aim at finding the most accurate disparity map [1, 2]. Generally, these methods are classified into two main categories, local and global methods. Local methods are of low computational complexity but are sensitive to some ambiguous regions such as occluded areas or regions with uniform textures. Global methods tend to be robust because they manipulate more information but are computationally more expensive than local methods. Whether local or global, these methods differ in the selected primitives (e.g. pixels, interest points, segments, blocks, edges), theirs attributes (e.g. gray level, color components, segment position), the cost function (including similarity measurements of two corresponding primitives), the constraints (e.g smoothness, ordering, uniqueness), the size of the matching window or the aggregation area. These methods are also related on different optimization criteria. Dynamic programming method is the oldest method [3, 4], at a pixel level, it sequentially finds the best matching given some constraints (smoothness, ordering and positiveness). The optimization may also rely on relaxation [5], graph cuts [6, 7], or belief propagation [8, 9].

Most papers concerned with stereoscopic image coding estimate block-based disparity maps since these maps can be encoded with lower bit-rates (i.e. each block is predicted with only one disparity-value). The main estimation technique is undoubtedly the Block-Matching Algorithm (BMA): each disparity is selected by minimizing a local distortion metric which is usually the sum of square differences or the sum of absolute differences [11], or less frequently correlation [10] and rank metrics [12]. All of these techniques are local and do not take into account choices of disparities elsewhere in the stereoscopic image.

In this paper, the block-based matching optimization approach proposed in [14] is extended to the case of non-rectified stereoscopic images. This approach is a global
method where the selected minimization criterion is a joint entropy-distortion metric. Entropy models the bit-rate that an entropy coding scheme would require to encode the disparity map. As a result, disparities are selected based not only on how the distortion is reduced but also on the disparity encoding bit-rate. The algorithm consists in processing each block in a raster scanning order and building sequentially a tree, of which only the $M$-best retained paths are extended at each depth in the tree.

The remainder of the paper is organized as follows. Section 2 reminds the principles of the Modified-M-Algorithm. It starts with the formalization of the rate-distortion optimization problem where the joint entropy-distortion metric is introduced. Followed by the description of the algorithm. Section 3 presents the modifications made to the MMA to adapt it to non-rectified stereoscopic images. Section 4 discusses the provided simulation results. Section 5 concludes our work.

2. BASICS CONCEPTS OF THE MMA

This section presents the block-based disparity map estimation algorithm for stereoscopic images. Before presenting the optimization approach, let us introduce some notations.

2.1. Notations

In what follows, the left view is considered as a reference view and the disparity map is estimated so as to provide a better prediction of the right view for a given bit-rate. $I_l$ and $I_r$ are respectively the left and the right views of size $K \times L$ of the stereoscopic image. Each view is composed of $X \times Y$ non-overlapped blocks of equal size $x \times y$. The pair $(i_p, j_p)$ represents the pixel-coordinates while $(i_b, j_b)$ the block-coordinates. The relation between these coordinates is: $i_p = i_b \times x$ and $j_p = j_b \times y$. These equations assume that the position of a block is given by the position of the pixel located at the top left corner of the block in the considered view. $I_r(i_p, j_p)$ (respectively $I_l(i_p, j_p)$) is the intensity of the pixel located at position $(i_p, j_p)$ in $I_r$ (respectively $I_l$). Images were initially assumed to be rectified so that the blocks are matched between the same scan lines. $\hat{I}_r$ is the predicted right view. The disparity, denoted $d(i_b, j_b)$, is associated with the block at position $(i_b, j_b)$ in the right image. Therefore all pixels within this block have the same disparity. Figure 1 summarizes these notations.

2.2. Rate-distortion optimization problem

The problem addressed in this paper concerns the estimation of the disparity map, denoted $d = \{d(i_b, j_b) \mid i_b = 0, ..., X - 1; j_b = 0, ..., Y - 1\}$, that minimizes the global distortion of the predicted right view:

$$E_{\text{Global}}(d) = \sum_{i_b=0}^{X-1} \sum_{j_b=0}^{Y-1} E_{\text{Block}}(i_b, j_b) \quad \text{with} \quad E_{\text{Block}}(i_b, j_b) = \sum_{u=0}^{x-1} \sum_{v=0}^{y-1} \left( \hat{I}_r(i_p + u, j_p + v) - I_l(i_p + u, j_p + v) \right)^2$$

where $\hat{I}_r(i_p+u, j_p+v) = I_l(i_p+u, j_p+v+d(i_p, j_p))$, (1)

subjected to an entropy constraint $H(d)$ which is an estimate of the bit-rate associated with the disparity map. This problem is formulated as a Lagrangian minimization:

$$\hat{d} = \text{argmin} J(\lambda, d) = \text{argmin} (E_{\text{Global}}(d) + \lambda H(d)), \quad (2)$$

where $\lambda$ is the Lagrange multiplier. Minimizing $J(\lambda, d)$ for any $\lambda$ gives the points on the convex hull of all possible Rate-Distortion (R-D) points.

2.3. The MMA, an entropy-constrained disparity map estimation algorithm

This section addresses the issue of estimating the disparity map associated with the right view of the stereoscopic image as formalized by equation (2) under the constraint of a low computational complexity.

To do so, the MMA exploits the underlying idea of the generic $M$-algorithm initially developed in communications field to estimate the transmitted data stream through a noisy channel using the maximum likelihood criterion [13]. In its original version, the $M$-algorithm sequentially built a tree. At each depth in the tree, the algorithm extends the $M$-best retained paths according to the maximum likelihood metric. Although the research domain is restricted, this sub-optimal optimization algorithm not only presents good performance
but also reduces the computational load of the optimization problem. However many changes were required to adapt this

generic $M$-algorithm to a stereoscopic block-based matching problem. These changes are shown in the following.

The blocks of the views are processed in a raster order and are numbered using 1D-coordinates. The depth of the tree is
denoted $t$. It corresponds to the position of the current block in the right view which is matched to the blocks in the left
view. This depth depends on the position $(i_b, j_b)$ of the block as follows:

$$ t = i_b \times Y + j_b \text{ with } i_b = 0, ..., X - 1 \text{ and } j_b = 0, ..., Y - 1. $$

(3)

The research area is reduced since a sliding matching window $W$ of size $1 \times N$, centered on the origin of the block of
the right view, is introduced. Moreover among the extended paths, only $M$-best paths have been retained according to a
joint entropy-distortion metric.

The different steps of the proposed optimization algorithm is developed in the following. Assume that our optimization
algorithm has already matched the blocks up to the block $t-1$ and that $M$-best paths have been retained according to their
joint entropy-distortion costs:

$$ J^{k}_{t-1}(\lambda, d) = E^{k}_{t-1} + \lambda H^{k}_{t-1} \quad \text{with } k = 1, ..., M, $$

(4)

where $E^{k}_{t-1}$ is the cumulative distortion metric and $H^{k}_{t-1}$ is the disparity entropy both associated with the $k$-th path at $(t-1)$-th depth in the tree. Each path is thus represented by a disparity map, denoted $S^k$, gathering $t-1$ disparities:

$$ S^k = \{d^{k}_{1}, d^{k}_{2}, ..., d^{k}_{t-1} \} \quad \text{with } k = 1, ..., M, $$

(5)

where $d^{k}_{t}$ is the disparity of the $k$-th path at $t$-th depth.

Consider now the next depth in the tree, i.e. $t$-th. Each $M$ paths is extended by $N$ branches where each branch has its
own disparity $w$ (with $w = w_{min}, ..., w_{max}$ such as $w_{max} - w_{min} + 1 = N$) and a local distortion $E^{w}_{bt}$ given by:

$$ E^{w}_{bt} = \sum_{u=0}^{x-1} \sum_{v=0}^{y-1} (f_r(i_p + u, j_p + v) - f_r(i_p + u, j_p + v))^2 $$

$$ \text{with } w = w_{min}, ..., w_{max}. $$

(6)

The distortion of each of the $M \times N$ extended paths is then updated:

$$ E^{m}_{bt} = E^{k}_{t-1} + E^{w}_{bt} \text{ for } m = 1, ..., M \times N $$

$$ \text{with } k = 1, ..., M \text{ and } w = w_{min}, ..., w_{max}. $$

(7)

For a given $\lambda$, the $J^{k}_{t}$ global cost of the $k$-th path at the $t$-th depth is computed as follows:

$$ J^{k}_{t}(\lambda, d) = E^{k}_{t} + \lambda H^{k}_{t} \quad \text{with } k = 1, ..., M \times N, $$

(8)

where $H^{k}_{t}$ the $k$-th disparity entropy, models the entropy derived from the true probability distribution of disparities of

the $k$-th path $(d^{k}_{1}, d^{k}_{2}, ..., d^{k}_{t})$:

$$ H^{k}_{t} = - \sum_{w=w_{min}}^{w_{max}} p^{k}_{t}(d = w) \log_2(p^{k}_{t}(d = w)) $$

for $k = 1, ..., M \times N$. \hspace{1cm} (9)

However this entropy cannot be calculated since it requires the true disparity distribution knowledge. The MMA estimates
these probabilities (i.e. $p^{k}_{t}(d = w)$) according to a finite mixture distribution represented as a sum of weighted
discrete distributions as follows:

$$ p^{k}_{t}(d = w|d^{k}_{1}, d^{k}_{2}, ..., d^{k}_{t}) = C_a \times p_a(d = w) $$

$$ + C_{emp} \times p_{emp}(d = w|d^{k}_{1}, d^{k}_{2}, ..., d^{k}_{t-1}) $$

$$ + C_c \times p_c(d = w|d = d^{k}_{t}), $$

(10)

where the coefficients $C_a, C_{emp}$ and $C_c$ satisfy the following condition: $C_a + C_{emp} + C_c = 1$, with $C_a = \frac{b}{b+a+b+c}$; $C_{emp} = \frac{b}{a+b+c}$ and $C_c = \frac{c}{a+b+c}$. These coefficients depend on the current depth, i.e. on the number of blocks processed and are parametrized as follows:

$$ a = X \times Y - t; \quad b = t \quad \text{and } c = 1. $$

(11)

$p_a$ is the probability density assumed to be a discrete uniform distribution on the selected matching window $W$ given by:

$$ p_a(d = w) = \frac{1}{N} $$

(12)

$\beta$ is a constant parameter smaller than 1. It provides a free parameter to adjust the weight of $p_a$ in the finite mixture
distribution (in equation (10)). The empirical probability $p_{emp}(d = w|d^{k}_{1}, d^{k}_{2}, ..., d^{k}_{t-1})$ is calculated from the retained
disparities until the $(t-1)$-th depth (i.e. $d^{k}_{1}, d^{k}_{2}, ..., d^{k}_{t-1}$). The probability $p_c(d = w|d = d^{k}_{t})$ is the probability related to the choice that the algorithm makes when it selects at depth $t$ the branch with disparity $w_c$ among the other branches:

$$ p_c(d = w|d = d^{k}_{t}) = \begin{cases} 1 \text{ if } w = w_c \\ 0 \text{ if } w \neq w_c \end{cases}. $$

(13)

Note that "$a$" decreases linearly while "$b$" increases linearly. Indeed as the image is being processed and as $t$ increases, $p_a$
is less and less needed. Therefore the estimation is more and more close to the true probability distribution.

The costs $J^{k}_{t}$, given by equation (8), are then computed and sorted in a decreasing order and the $M$-best paths are re-
tained. The $M$ disparity maps (i.e. $S^k$) are also updated. This process is iterated until processing the last block of the right
view. At that point, the first path contains the best disparity map in terms of entropy-distortion.

The optimization algorithm is summarized in Figure 2 and an illustration is given by Figure ?? using $M = 2$ and $N = 5$. 

3. EXTENSION OF THE MMA TO THE CASE OF NON-RECTIFIED STEREOCOPIC IMAGES

In this section, the modifications brought to the MMA so that it can be used also for non-rectified stereoscopic images are presented. First, we need to redefine some notations. The blocks to be matched do not necessarily belong to the same scan lines, so that disparity vectors have now two components. Let \( d(i_b, j_b) = (d_x, d_y) \) be the disparity associated to the block at position \((i_b, j_b)\) in the right image, \(d_x\) being its vertical component and \(d_y\) its horizontal component as represented in Figure 3. The rate-distortion optimization problem introduced in section 2.2 remains unchanged. However, the expression of the global distortion \( E_{\text{global}}(d) \) of the predicted image is adapted to take into account the two components of the disparity vectors:

\[
E_{\text{global}}(d) = \sum_{i_b=0}^{X-1} \sum_{j_b=0}^{Y-1} E_{\text{Block}}(i_b, j_b) \quad \text{with} \quad E_{\text{Block}}(i_b, j_b) = \sum_{u=0}^{X-1} \sum_{v=0}^{Y-1} (I_r(i_p + u + d_x, j_p + v + d_y) - I_r(i_p + u, j_p + v))^2
\]

(14)

To match the blocks of the two views, the MMA relies on a sliding matching window which is now extended to consider also vertical displacements. Let \( W \) be the window of size \( N' = W_x \times W_y \) centered on the origin of the block to be matched in the right view as shown in Figure 3. The different steps of the MMA remains unchanged, but some modifications have been made to the MMA to take into account choices of 2-D disparity vectors. These changes are listed below:

- At each depth of the tree, the \( M \) best retained paths are extended by \( N' \) branches. Each branch is associated to a choice of disparity \( w = (w_x, w_y) \) where \( w_x = \{ w_{x,\min}, \ldots, w_{x,\max} \} \) and \( w_y = \{ w_{y,\min}, \ldots, w_{y,\max} \} \) (such as \( w_{x,\max} - w_{x,\min} + 1 = W_x \) and \( w_{y,\max} - w_{y,\min} + 1 = W_y \)).
- The local distortion \( E_{\text{bt}}(w) \) of each branch is computed as:
  \[
  E_{\text{bt}}(w) = \sum_{u=0}^{x-1} \sum_{v=0}^{y-1} (\hat{I}_r(i_p + u, j_p + v) - I_r(i_p + u, j_p + v))^2
  \]
where \( \hat{I}_r(i_p + u, j_p + v) = I_l(i_p + u + w_x, j_p + v + w_y) \),
(15)
- The probability \( p_a(d = w) \) is modified to compute the entropy associated to each path according to:
  \[
  p_a(d = w) = \frac{1}{N'}
  \]
(16)

4. EXPERIMENTAL RESULTS

This section discusses the performance of the extended MMA. The Peak Signal-to-Noise Ratio (PSNR) is evaluated between the original and the predicted right views versus the approximated bit-rate given by the entropy of the empirical distribution of the estimated disparity map converted into bit per pixel (bpp).

Simulation results are compared to the traditional BMA which finds the best match for each block by minimizing the Sum of Square Differences (SSD). The provided simulations have been performed on stereoscopic images from Middlebury and CMU stereo dataset [15, 16].
Figure 6 shows the performance of the BMA on the stereoscopic image "Sand" from [16], taking blocks of size 8 × 8. Experiments have been conducted using several sizes of symmetrical windows, $W_x \times W_y$, centered on the origin of the block to be matched (as shown in Figure 3), with $W_x = \{1, 3, 5, \ldots, 41\}$ and $W_y = \{1, 3, 5, \ldots, 301\}$. For each size of windows, a disparity map is estimated by the BMA and the quality of the predicted image using this map is measured with the PSNR. Each point of the figure 6 corresponds to the performance of the BMA using a given size of window: the pixel at position $(i, j)$ (with $i = \{0, 1, 2, \ldots, 20\}$ and $j = \{0, 1, 2, \ldots, 150\}$) represents the PSNR achieved using a window of size $W_x \times W_y = (i \times 2 + 1) \times (j \times 2 + 1)$. Values of PSNR obtained by varying the window size are represented using a grayscale where black color corresponds to the lowest quality achieved by the BMA (14dB) and the white color corresponds to the highest quality (24dB). It can be noticed that in this experiment, increasing the vertical component $W_y$ of the window also increase the PSNR of the predicted image up to a certain point. When increasing $W_y$ beyond 20, no significant gain is achieved in the quality of the reconstructed image.

In the next experiment performed on the same stereoscopic image, window with vertical component $W_y \geq 20$ are not considered. Figure 5 depicts the performance of the BMA and the extended MMA using blocks of size $8 \times 8$ and $10 \times 10$ (represented respectively with the red dashed curves and the blue solid curves), and several window sizes $(1 \times 30, 10 \times 30$ and $20 \times 30$ represented respectively using 'o', 'x' and '+' symbols). The performance of the BMA is given by the very first point on the top right of each curves, which also corresponds to the performance achieved by the extended MMA taking $\lambda=0$, as both algorithms only intend to minimize the distortion of the predicted image. From all of these points starts a curve representing the performance of the extended MMA plotted for several values of $\lambda$. Using a window of size $1 \times 30$ and blocks of size $8 \times 8$, the BMA gives the R-D point (14.6dB, 0.07bpp), while the MMA can still reach this quality of reconstruction for a bitrate much lower (0.02bpp) for a given value of $\lambda$, thus allowing a gain of 71% for the same PSNR. By extending vertically the searching window, which is more adapted to this non-rectified stereoscopic image, the BMA yields to the R-D point (21.4dB, 0.14bpp). Allowing a little decrease of the predicted image quality (21.1dB), the extended MMA reduces the bitrate to 0.03bpp (corresponding to a decrease of 79%). It can also be noted that using blocks of size $10 \times 10$, the BMA gives a predicted image quality of 21.2dB for a bitrate of 0.09bpp. For the same bitrate, the extended MMA achieves a gain of 0.2dB over the BMA using smaller blocks of size $8 \times 8$.

Figure 7 shows the performance of the BMA and the MMA on the stereoscopic image "Tsukuba" from Middlebury dataset, using blocks from $4 \times 4$ to $8 \times 8$, and a window of size $3 \times 30$. Performance of the BMA are given by the circles joined by the red dashed line, each circle corresponding to a different size of block. From each of these circles starts a blue curve (in solid line) representing the performance of the extended MMA for the same size of block and several values of $\lambda$. The BMA gives a reconstructed image of quality 25.3dB for a bitrate of 0.36bpp using blocks of size $4 \times 4$. The extended MMA allows to reduce the bitrate up to 0.19bpp for a just little lower PSNR of 25.2dB. The extended MMA also achieves a gain in the quality of the predicted image for the same bitrate. As an example, at 0.15bpp, the BMA gives a PSNR of 24.2dB for the predicted image while for the same bitrate, the extended MMA yields to 25.0dB, resulting in a gain of 0.8dB. Simulations on other stereoscopic images confirm the benefits of our approach with respect to the BMA.

5. CONCLUSION

An extension of the disparity map estimation algorithm called MMA has been presented for non-rectified images. The original MMA is related to the generic $M$-algorithm which consists to sequentially build a tree where only $M$-best paths are retained at each depth. The optimization algorithm is based on a joint entropy-distortion metric. The extended MMA still ensure a better predicted image in terms of PSNR while also reducing the bitrate required to encode the estimated disparity map compared to the traditional BMA in the case of non-rectified stereoscopic images.

Fig. 4. Rate-distortion optimization on "Wood2".
Fig. 5. Rate-distortion optimization on "Sandwich".

Fig. 6. Rate-distortion optimization on "Wood2".

Fig. 7. Rate-distortion optimization on "Baby1".

6. REFERENCES


