# ENTROPY-CONSTRAINED DENSE DISPARITY MAP ESTIMATION ALGORITHM FOR STEREOSCOPIC IMAGES

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## ABSTRACT

This paper deals with the stereo matching problem to estimate a dense disparity map. Traditionally a matching metric such as mean square error distortion is adopted to select the best matches associated with disparities. However several disparities related to a given pixel may satisfy the distortion criterion although quite often the choice that is made does not necessarily meet the coding objective. An entropy-constrained disparity optimization approach is developed where the traditional matching metric is replaced by a joint entropy-distortion metric so that the selected disparities reduce not only the disparity entropy but also the reconstructed image distortion. The algorithm sequentially builds a tree avoiding a full search and ensuring good rate-distortion performance. At each tree depth, the $M$ best retained paths are extended to build new paths which are assigned entropy-distortion metrics. Simulations show that our algorithm provides better results than dynamic programming algorithm.

***Index Terms*—** Stereoscopic images, matching, disparity, entropy, optimization

## 1. INTRODUCTION

Stereoscopic image (3D image) is composed of an image pair captured by a stereo camera which provides the same scene for the right and left eye. Hence it requires twice amount of information to be transmitted or stored compared to a traditional image (2D image). Therefore to compress efficiently stereoscopic image, the developed coding algorithms exploit inter-view redundancies since the stereo camera captures the same scene. The underlying idea consists to extract the spatial displacements between the left and right image to estimate the disparity map. Its efficient estimation ensures a reliable reconstruction of the original stereoscopic image.

The state of the art performed on the estimating problem of the disparity map shows that many studies already addressed this problem and several stereoscopic matching algorithms have been deployed. The stereoscopic matching problem is generally formulated as the minimization problem over the overall image of an energy function (global cost) resulting in global approaches or several energy functions (local costs) resulting in local approaches. These methods, as summarized in [1, 2], differ in the choice of : (i) the primitives (e.g. pixels, interest points, segments, regions, edges) and their attributes (e.g. gray level, contrast, color components, segment position, segment orientation) to be matched ; (ii) the global cost of correspondence including the local matching costs which measure the dissimilarity between two corresponding primitives and the constraint cost for all correspondences (e.g. uniqueness, ordering, smoothness); (iii) the matching window size; (iv) the aggregation area ; and (v) the optimization method.

The main objective of the optimization methods is to minimize the global or local cost to ensure a high matching accuracy. The most naive method is the greedy search which performs an exhaustive search for the best matching. For its high computational load, this method has not been retained. Dynamic programming technique has been the first optimization method introduced in stereo matching context where smoothness constraints have been added to optimize matches in scan lines [3]. However among different developed versions, Veksler imposed smoothness in both horizontal and vertical directions with the objective of recovering the real disparity map [4]. Many other optimization methods such as relaxation [5], graph cut [6, 7] and believe propagation [8, 9] have been also exploited.

Among the stereoscopic matching approaches developed in literature, this paper focuses on pixel-based matching algorithms. Mean Square Error (MSE) and Mean Absolute Error (MAD) are usually used as matching criterion. Sometimes, for a given pixel it is possible to get not only one correspondence but also a set of correspondences satisfying the criterion. Nevertheless, some solutions are more expensive than others in terms of bit-rate. To address this problem, we propose to replace the traditional matching metric by a joint entropy-distortion metric so that the selected disparities reduce not only the predicted image distortion but also the disparity entropy. This problem is formalized by the Lagrangian minimization where the cost function is exploited as the new matching metric. To avoid computational load related to a full search solutions, we rely on a tree which is sequentially constructed. At each tree depth, the algorithm retains the $M$ best paths which will be extended in the next step.

The remainder of the paper is organized as follows. Section 2 first introduces notations and assumptions, then states the optimization problem which is formulated by the Lagrangian minimization. The proposed entropy-constrained dense disparity map algorithm is then developed. Section 3 discusses simulation results. Section 4 concludes the work.

## 2. PROPOSED ALGORITHM

Assumptions and notations are introduced before describing the proposed optimization algorithm. Images of the left and right view of the stereoscopic image are assumed to be rectified. $I_l$ and $I_r$ represent respectively the left and right image of size $K \times L$. $I_r(i,j)$ (respectively $I_l(i,j)$) is the intensity of the pixel located at position $(i,j)$ in $I_r$ (respectively $I_l$).

In what follows, the proposed algorithm is described so that it estimates the disparity map related to the right view $I_r$ using the left view $I_l$ as reference image.

### 2.1. Rate-distortion optimization problem

The problem addressed in this paper concerns the estimation of the disparities field that minimizes the global distortion cost of the reconstructed right image expressed by:

$$E_{global} = \sum_{i=0}^{K-1} \sum_{j=0}^{L-1} (\widehat{I}_r(i,j) - I_r(i,j))^2$$
$$\text{with } \widehat{I}_r(i,j) = I_l(i, j + d(i,j)), \quad (1)$$

subject to a given entropy constraint $H$ (i.e. number of bits spent to encode a disparity). $d(i,j)$ is the spatial displacement associated with the pixel $I_r(i,j)$.

This problem is formulated by the Lagrangian minimization which consists to find the points on the convex hull of all possible Rate-Distortion (R-D) points:

$$J_{min}(\lambda, d) = min(E_{global} + \lambda H), \quad (2)$$

where $\lambda$ is the Lagrange multiplier and $d$ the disparities field.

### 2.2. Entropy-constraint dense disparity map based on M-algorithm

This section deals with the optimization problem formulated by equation (2) where the main objective is to estimate an efficient dense disparity map associated with one view of the stereoscopic image in terms of entropy-distortion.

The underlying idea of the developed algorithm is related to the generic sequential decoding $M$-algorithm deployed in [10]. This algorithm has been also exploited in communications to estimate the transmitted data stream through a noisy channel according to the maximum likelihood criterion. This algorithm is a sub-optimal optimization method based on a tree-search technique parsing only a part of the tree. However many changes have been made to this algorithm to be adapted to stereo matching problem.

To reduce the computational load of the optimization problem, the proposed algorithm limits the matching search process while trying to ensure good performance in terms of entropy-distortion. A sliding matching window $W$ of size $N$ centered on the pixel located at position $(i,j)$ on the right image $I_r$ is introduced.

The $t$-th depth of the tree depends on the row index $i$, the column index $j$ and the number of columns of the image $I_r$. It is expressed as follows:

$$t = i \times L + j \text{ with } i = 0, ..., K-1 \text{ and } j = 0, ..., L-1. \quad (3)$$

At $(t-1)$-th depth of the tree, assume that the $M$ best retained paths are sorted in a decreasing order according to the entropy-distortion cost $J^k_{t-1}(\lambda, d)$ given by:

$$J^k_{t-1}(\lambda, d) = E^k_{t-1} + \lambda H^k_{t-1} \ \text{ with } k = 1, ..., M, \quad (4)$$

where $E^k_{t-1}$ is the cumulative distortion metric and $H^k_{t-1}$ is the disparity entropy both associated with the $k$-th path at $(t-1)$-th depth. At this stage $M$ best disparity maps, denoted $S^k$, are retained:

$$S^k = \{d^k_1, d^k_2, ..., d^k_{t-1}\} \ \text{ with } \ k = 1, ..., M, \quad (5)$$

where $d^k_l$ is the disparity of the $k$-th path at $l$-th depth.

On the next depth, i.e. $t$-th, each $M$ selected path is then extended by $N$ branches. Each branch is affected by a disparity equal to $w$ (with $w = -n, ..., n$ depending on the sliding matching window $W$) and a local distortion ($E_{b_t}^{w}$) given as follows:

$$E_{b_t}^{w} = ((I_l(i,j) - I_r(i, w+j))^2 \text{ with } w = -n, ..., n. \quad (6)$$

The distortion of each of the $M \times N$ extended paths is then updated according to:

$$E^m_t = E^k_{t-1} + E_{b_t}^{w} \text{ for } m = 1, .., M \times N$$
$$\text{with } k = 1, ..., M \text{ and } w = -n, .., n. \quad (7)$$

For a given $\lambda$, the $J^k_t(\lambda, d)$ cost on the $t$-th depth is computed as given below:

$$J^k_t(\lambda, d) = E^k_t + \lambda H^k_t \text{ with } k = 1, ...M \times N, \quad (8)$$

where $H^k_t$ is the entropy associated with disparities (i.e. $d^k_1, d^k_2, ..., d^k_t$) of the $k$-th path at $t$-th depth provided by

$$H^k_t = - \sum_{w=-n}^{n} p^k_t(d = w) log_2(p^k_t(d = w))$$
$$\text{for } k = 1, ...M \times N. \quad (9)$$

However this entropy can not be calculated since it requires the probability distribution knowledge of disparities. For this, we propose to estimate these probabilities (i.e. $\{p_t^k(d = w)\}$) according to a finite mixture distribution represented as a sum of weighted discrete distributions as follows:

$$\begin{aligned} \widehat{p}_t^k(d = w|d_1^k, d_2^k, ..., d_t^k) = C_a \times p_a(d = w) + \\ C_b \times p_{exp}^k(d = w|d_1^k, d_2^k, ..., d_{t-1}^k) + \\ C_c \times p_c(d = w|d = d_t^k), \end{aligned} \quad (10)$$

where the coefficients $C_a$, $C_{exp}$ and $C_c$ satisfy the following condition:

$$C_a + C_{exp} + C_c = 1, \quad (11)$$

with $C_a = \beta \times \frac{a}{a+b+c}$; $C_{exp} = \frac{b}{a+b+c}$ and $C_c = \frac{c}{a+b+c}$. These coefficients depend on the current depth, i.e. on the number of pixels processed and are parameterized as follows:

$$a = K \times L - (i \times L + j);\ b = i \times L + j \text{ and } c = 1. \quad (12)$$

$\beta$ is a constant parameter smaller than 1. It provides a freedom degree to adjust the first probability density (in equation (10)) assumed to be a discrete uniform distribution on the selected matching window given by:

$$p_a(d = w) = \frac{1}{2n+1} = \frac{1}{N} \text{ with } w = -n, ..., n. \quad (13)$$

The probability $p_{exp}^k(d = w|d_1^k, d_2^k, ..., d_{t-1}^k)$ is calculated from the retained disparities until the $(t-1)$-th depth (i.e. $d_1^k, d_2^k, ..., d_{t-1}^k$).

The probability $p_c(d = w|d = d_t^k)$ is the probability related to the choice that the algorithm makes when it selects at depth $t$ the branch with disparity $w_c$ among other branches:

$$p_c(d = w|d_t^k = w_c) = \begin{cases} 1 \text{ if } w = w_c \\ 0 \text{ if } w \neq w_c \end{cases} \quad (14)$$

The $J_t^k$ costs are then sorted in a decreasing order and $M$ best paths are retained. The $M$ disparity maps (i.e $S^k$) are also updated. This process is iterated until scanning the complete reference image. Therefore the first path contains the best disparity map in terms of entropy-distortion. Figure 1 illustrates an example for $M = 2$ and $N = 5$.

The different steps of the proposed optimization algorithm are summarized below.

Algorithm 1: Entropy-distortion $M$-algorithm

**Input:** Left image $I_l$ and right image $I_r$ of size $K \times L$
**Output:** Estimated dense disparity map associated with $I_r$;

**1.** Set initial values: $\lambda$; $M$; $N$; $\beta$; $i = -1$ and $j = -1$;
**2.** Increment by 1 the row index $i$;
**3.** Increment by 1 the column index $j$;
**2.** Set the sliding matching window on the pixel $I_r(i, j)$;
**3.** Extend all $M$ best current paths of the tree to depth $t$;
**4.** Compute the distortions of $M \times N$ branches;
**5.** Update the distortions of the extended paths;
**6.** Estimate the disparity probabilities of each path;
**7.** Deduce the disparity entropy of each path;
**8.** Compute the entropy-distortion costs of each path;
**9.** Sort the paths in a decreasing order of rate-distortion cost;
**10.** Select among the $M \times N$ paths, the $M$ best paths;
**11.** Update the $M$ disparity maps;
**12.** Start again from step 3 if $j < L$ otherwise continue;
**12.** Start again from step 2 if $i < K$ otherwise continue;
**13.** Select the best dense disparity map associated with $I_r$.

## 3. SIMULATION RESULTS

In this section, simulation results are presented to evaluate the performance of the proposed optimization algorithm. Comparisons are carried out with the dynamic programming algorithm provides by the computer vision system toolbox of Matlab [12]. This algorithm exploits not only block matching metric as the cost function but also constrains the disparities to change very slightly between adjacent pixels. Middlebury stereo dataset is used [11].

Simulations provided in this paper are performed on Poster stereoscopic image using im6.ppm (respectively im2.ppm) for the right (respectively left) view presented by Figure 2. The spatial resolution of these images is equal to 383×435 pixels. In the provided results, the right view is reconstructed. The performance in terms of distortion is measured in terms of $PSNR$ calculated between the original and reconstructed images using the luminance component.

Figure 3 concerns the reconstructed right view using our estimated disparity map with the following parameters: $M = 4$; $\beta = 0.04$; $\lambda = 200000$ and $N = 30$. The evaluated $PSNR$ is equal to 29.31dB with a bit-rate equal to 3.47bpd (bits per disparity). Figure 4 is related to the reconstructed right view according to dynamic programming disparity map using the same window size (i.e. $N = 30$) as in our algorithm. The evaluated $PSNR$ is equal to 23.17dB with 3.51bpd. For an equivalent bit-rate (3.47ppd and 3.51ppd), we obtain a gain of 6dB in terms of $PSNR$. This is confirmed by what we observe on the reconstructed images. Indeed we clearly see that the reconstruction of ”newspaper” part in the Poster image is of lower quality for dynamic programming disparity map.

Figure 6 (respectively Figure 5) shows the estimated disparity map used to reconstruct the right view given by Figure 3 (respectively Figure 4). The distribution of disparities is completely different and is provided by Figures 8 and 9.

Figure 11 provides the rate-distortion optimization given for different values of $\lambda$ with the following parameters: $M = 4$; $\beta = 0.04$ and $N = 30$. Even if the bit-rate involved by the dynamic programming technique is divided by 2 (i.e. 1.67bpd), a gain of more than 3dB in terms of $PSNR$ (26.55dB) can still be reached. For bit-rates up to at least 1.1bpd, the reconstruction quality is still better compared to that obtained with dynamic programming disparity map.

For low entropies, without applying any smoothness constraint, the estimated disparity map is nevertheless smooth. An example is provided by Figure 7 in which the entropy is equal to 0.9bpd. This is also confirmed by the disparity distribution illustrated by Figure 10. Therefore for future investigations, it would be interesting to modify the estimated dense disparity map using a relevant division into blocks so as to increase the performance of our optimization algorithm. Another track would be to adapt our algorithm so that the entropy would be calculated on the difference between consecutive disparities rather than on disparities.

## 4. CONCLUSION

This paper addressed stereo matching problem where a pixel-based approach is adopted to estimate a dense disparity map. The optimization problem of selecting the best disparities in terms of entropy-distortion is formulated by the Lagrangian minimization. In order to reduce the computational load associated with a full search solutions, this optimization problem statement is solved according to the developed algorithm which sequentially builds a tree avoiding a full search and ensuring good rate-distortion performance. Indeed at each tree depth, only the $M$ best retained paths are extended to build new paths for which entropy-distortion metrics are assigned. Simulations performed on stereoscopic images clearly show the advantage of our algorithm compared to the dynamic programming technique in the particular context of coding.

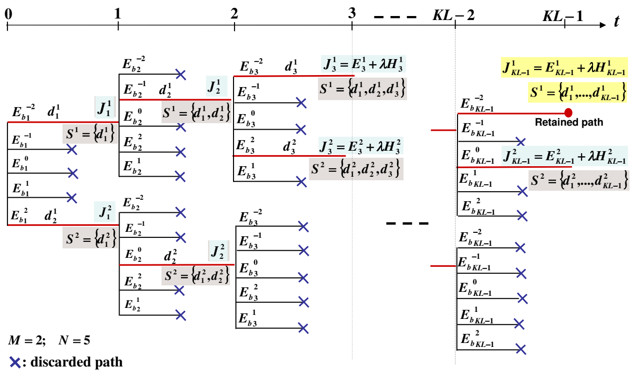


**Fig. 1**. Entropy-constrained dense disparity map estimation algorithm with $M = 2$ and $N = 5$.

**Fig. 2**. Original right image.

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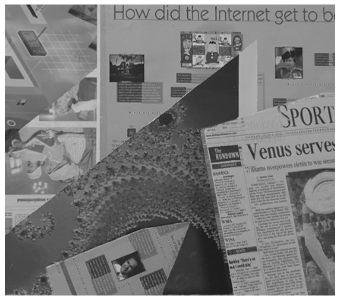

**Fig. 3**. Reconstructed image using our estimated disparity map ($PSNR$=29.31dB and $H$= 3.47bpd).

**Fig. 4**. Reconstructed image using dynamic programming disparity map ($PSNR$=23.17dB and $H$=3.51bpd).

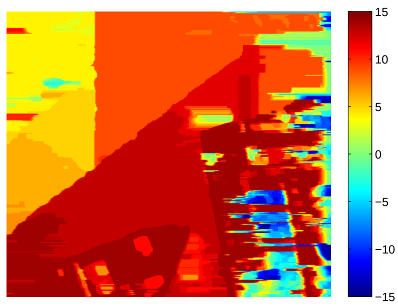


**Fig. 5**. Disparity map using dynamic programming (with 23.17dB; 3.51bpd).

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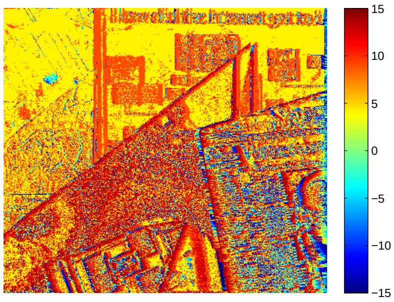

**Fig. 6**. Estimated disparity map (3.47bpd; 29.31dB).

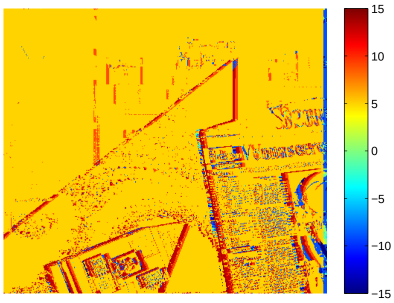

**Fig. 7**. Estimated disparity (0.9bpd; 22.04dB).

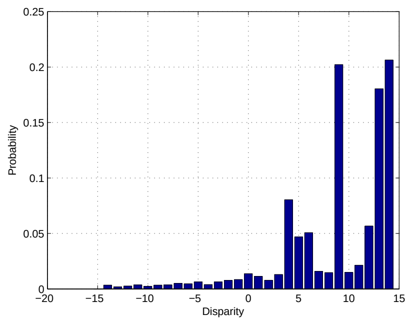


**Fig. 8**. Disparity distribution for dynamic programming.

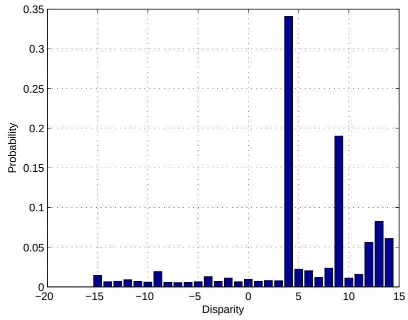


**Fig. 9**. Disparity distribution for the proposed algorithm (with 3.47bpd).

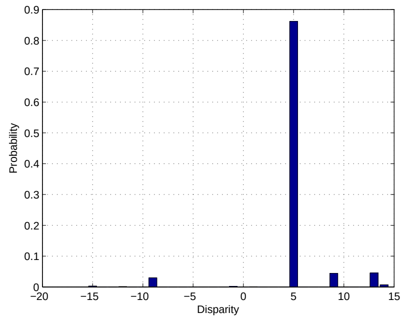


**Fig. 10**. Disparity distribution for the proposed algorithm (with 0.9bdp).

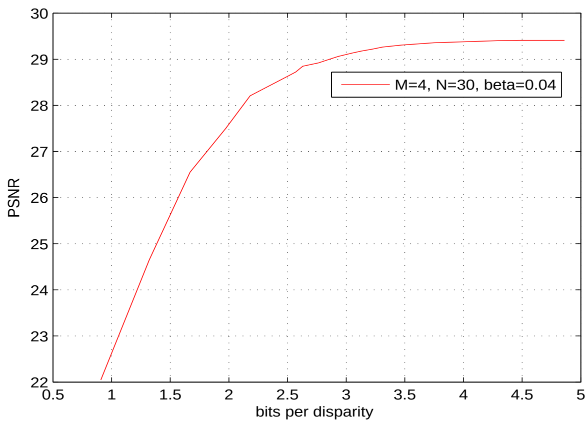


**Fig. 11**. Rate-Distortion optimization.


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