Séance 11

Exercices

exercice 1

1. $y(t) = h(t) * \mathbb{1}_{[-1/2,1/2]}(t) = \int_{-\infty}^{+\infty} e^{-\pi\tau^2} \mathbb{1}_{[-1/2,1/2]}(t-\tau)\, d\tau$

$y(t) = \int_{-\infty}^{+\infty} e^{-\pi\tau^2} \mathbb{1}_{[t-1/2,\, t+1/2]}(\tau)\, d\tau$

$y(t) = \int_{t-1/2}^{t+1/2} e^{-\pi\tau^2}\, d\tau.$

2. $\int_{-\infty}^{+\infty} e^{-t^2} dt = \int_{-\infty}^{+\infty} e^{-\frac{t^2}{2\left(\frac{1}{\sqrt{2}}\right)^2}} dt = \sqrt{2\pi} \times \frac{1}{\sqrt{2}} = \sqrt{\pi}.$

e^{-t^2} est pair donc $\int_0^{+\infty} e^{-t^2} dt = \frac{1}{2}\sqrt{\pi}.$

$\frac{2}{\sqrt{\pi}} \int_0^{+\infty} e^{-t^2} dt = 1.$

donc $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^{x} e^{-t^2} dt \to 1.$

Sa, Er4

exercice 3

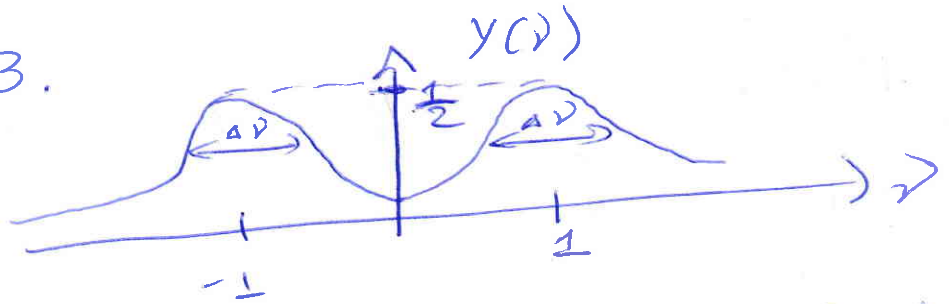
1. $X(\nu) = \frac{1}{2}\delta(\nu-1) + \frac{1}{2}\delta(\nu+1)$

2. $y(t) = x(t)\, w(t)$

$Y(\nu) = X(\nu) * W(\nu)$

$Y(\nu) = \frac{1}{2} W(\nu-1) + \frac{1}{2} W(\nu+1)$

3.



Pour Déterminer $\Delta\nu$, $Y(1+\frac{\Delta\nu}{2}) = \frac{1}{2} e^{-\pi(\frac{\Delta\nu}{2})^2} = \frac{1}{4}$

$\left(\frac{\Delta\nu}{2}\right)^2 = \frac{\ln 2}{\pi}$

$\Delta\nu = 2\sqrt{\frac{\ln 2}{\pi}}$

S11, Ex 2

3. $$y(t)=\int_{t-1/2}^{t+1/2} e^{-\pi z^2}\,dz = \int_{\sqrt{\pi}(t-1/2)}^{\sqrt{\pi}(t+1/2)} e^{-z'^2}\frac{dz'}{\sqrt{\pi}}$$

$$z'=\sqrt{\pi}\,z \qquad dz'=\sqrt{\pi}\,dz$$

$$y(t) = \frac{1}{2}\times \operatorname{erf}\left(\sqrt{\pi}(t+1/2)\right) - \frac{1}{2}\operatorname{erf}\left(\sqrt{\pi}(t-1/2)\right)$$

4. $$y(t)=\frac{1}{2}\left(1-\operatorname{erf}\left(\sqrt{\pi}(t-1/2)\right)\right) - \frac{1}{2}\left(1-\operatorname{erf}\left(\sqrt{\pi}(t+1/2)\right)\right)$$

$$\frac{1-\operatorname{erf}\left(\sqrt{\pi}(t+1/2)\right)}{1-\operatorname{erf}\left(\sqrt{\pi}(t-1/2)\right)} = \frac{1-\operatorname{erf}\left(\sqrt{\pi}(t+1/2)\right)}{\dfrac{e^{-\pi(t+1/2)^2}}{(t+1/2)\sqrt{\pi}\sqrt{\pi}}} \times \frac{e^{-\pi(t-1/2)^2}}{(t-1/2)\sqrt{\pi}\sqrt{\pi}} \times \frac{1}{1-\operatorname{erf}\left(\sqrt{\pi}(t-1/2)\right)}$$

$$\times \frac{e^{-\pi(t+1/2)^2}}{(t+1/2)\sqrt{\pi}\sqrt{\pi}} \times \frac{(t-1/2)\sqrt{\pi}\sqrt{\pi}}{e^{-\pi(t-1/2)^2}} \to 0.$$

Donc

$$\frac{y(t)}{\frac{1}{2}\left(1-\operatorname{erf}\left(\sqrt{\pi}(t-1/2)\right)\right)} \to 1.$$

5. $$\frac{\frac{1}{2}\left(1-\operatorname{erf}\left(\sqrt{\pi}(t-1/2)\right)\right)}{\frac{1}{2}\times\dfrac{e^{-\pi(t-1/2)^2}}{\sqrt{\pi}\sqrt{\pi}(t-1/2)}} \to 1$$

Donc $$\frac{y(t)}{\dfrac{e^{-\pi(t-1/2)^2}}{\pi t}} \to 1.$$ qui est une asymptote de $y(t)$ en $+\infty$.

<u>exercice 2</u>

1. $H(0) = \int_{-\infty}^{+\infty} h(t)\, e^{-2i\pi 0 t} dt = \int_{-\infty}^{+\infty} h(t) dt$

$H(0) = \int_{-\infty}^{+\infty} e^{-\frac{t^2}{2\times\left(\frac{1}{\sqrt{2}\sqrt{\pi}}\right)^2}} dt = \sqrt{2\pi} \times \frac{1}{\sqrt{2}\sqrt{\pi}} = 1$

$H(0) = a = 1$.

2. $h(0) = \int_{-\infty}^{+\infty} H(\nu)\, e^{+2i\pi 0 \nu} d\nu = \int_{-\infty}^{+\infty} H(\nu) d\nu$

$h(0) = \int_{-\infty}^{+\infty} e^{-bt^2} dt = \int_{-\infty}^{+\infty} e^{-\frac{t^2}{2\left(\frac{1}{\sqrt{2}\sqrt{b}}\right)^2}} dt$

$= \sqrt{2\pi} \times \frac{1}{\sqrt{2}\sqrt{b}} = 1$ Donc $b = \pi$.

3. $X(\nu) = \frac{1}{2}\delta(\nu-1) + \frac{1}{2}\delta(\nu+1)$

Car $TF^{-1}[X(\nu)] = \frac{1}{2} e^{-i2\pi t} + \frac{1}{2} e^{2i\pi t}$

$= \cos(2\pi t) = x(t)$.

$Y(\nu) = H(\nu)X(\nu) = \frac{H(1)}{2}\delta(\nu-1) + \frac{H(-1)}{2}\delta(\nu+1)$

$Y(\nu) = \frac{e^{-\pi}}{2}\delta(\nu-1) + \frac{e^{-\pi}}{2}\delta(\nu+1)$

4. $TF^{-1}[\delta(\nu-1)] = e^{2i\pi t}$

$TF^{-1}[\delta(\nu+1)] = e^{-2i\pi t}$

$y(t) = \frac{e^{-\pi}}{2} e^{2i\pi t} + \frac{e^{-\pi}}{2} e^{-2i\pi t}$

$= e^{-\pi}\cos(2\pi t)$.