Vector Quantization for Image Compression based on Fuzzy Clustering

Abdel-Ouahab BOUDRAA, Qosai KANAFANI, Azedine BEGHDADI and Anissa ZERGAINOH

L2TI, Institut Galilée, Université Paris XIII
Avenue J.B. Clément, 93430 Villetaneuse, France
e-mail: abdel.boudraa@l2ti.univ-paris13.fr

Abstract

In this paper a codebook designing for image compression based on the Fuzzy c-Means (FCM) algorithm is presented. The codebook designing from training vectors is viewed as fuzzy clustering problem of unlabeled data points into clusters. Due to computational cost of FCM to generate the codebook, a Fast version (FFCM), which operates on the image histogram, is used to obtain a good initial codebook to start the FCM algorithm. Experimental results are presented to illustrate the performance of the proposed compression method.

1. Introduction

Image compression is the process of reducing the number of bits required to represent an image. Vector Quantization (VQ) is a data compression method where a set of data points is encoded by a reduced set of reference vectors, the codebook. VQ has been shown to be useful in compressing data that arises in wide range applications, including image processing [1], [2], speech processing [3], facsimile transmission [4] and weather satellites [5]. From rate distortion theory, it can be shown that VQ can achieve better compression performance than any conventional coding technique which is based on the encoding of scalar quantities [1]. The aim of the VQ is to design a codebook. An optimal VQ system is one that uses a codebook that yields the least average distortion, of the reconstructed image, among all possible codebooks. Different methods for designing a codebook have been proposed in [6]-[9]. The most popular one is the LBG algorithm [6]. In this work, the codebook designing from training vectors is viewed as a clustering problem of unlabeled data points into clusters.

In the standard approach, most clustering algorithms assume that each pixel (or voxel) belongs to single cluster. However, in general, the image regions are not always crisply defined. Image ambiguity within pixels is due to the possible multi-valued levels of brightness in the image. This indeterminacy is due to inherent vagueness rather than randomness. Thus, it is convenient and appropriate to regard the regions as fuzzy subsets of the image [10]. The fuzzy subsets are characterized by the possibility (degree) of belonging of each pixel to them. One popular method for assigning multi-subset membership values to pixels, for either segmentation or other types of processing, is the Fuzzy c-Means (FCM) algorithm [11].

2. Fuzzy clustering technique

Let $X = \{x_1, \ldots, x_n\}$ be the training set and $c \geq 2$ an integer. $x_k \in \mathbb{R}^S, \forall k = 1, 2, \ldots, n$ is a training vector from $S$-dimensional Euclidean space and let $\mathbb{R}^{c \times n}$ denotes the set of all real $c \times n$ matrices. A fuzzy $c$-partition of $X$ is represented by a matrix $U = [\mu_{ik}] \in \mathbb{R}^{c \times n}$, the entries of which satisfy

$$\mu_{ik} \in [0, 1] \quad 1 \leq i \leq c, \quad 1 \leq k \leq n \quad (1)$$

$$\sum_{i=1}^{c} \mu_{ik} = 1 \quad 1 \leq k \leq n \quad (2)$$

$$\sum_{k=1}^{n} \mu_{ik} > 0 \quad 1 \leq i \leq c \quad (3)$$

$U$ can be used to describe the cluster structure of $X$ by interpreting $\mu_{ik}$ as the degree of membership of $x_k$ to cluster $i$. Good partitions $U$ of $X$ may be defined by the minimization of the fuzzy $c$-means objective functional. The codebook vectors are evaluated by minimizing the distortion measure (4):

$$J_m(U, V : X) = \sum_{k=1}^{n} \sum_{i=1}^{c} (\mu_{ik})^m || x_k - v_i ||_A^2 \quad (4)$$

where $m \in [1, +\infty[$ is a weighting exponent called the fuzzifier, $V = (v_1, v_2, \ldots, v_c)$ is the vector of the cluster centers: codebook of size $c$. $\| x \|_A = \sqrt{x^T A x}$ is any inner product norm where $A$ is any positive definite matrix. Approximate optimization of $J_m$ by
the FCM algorithm is based on iteration through the following necessary conditions for its local extrema: FCM Theorem [11]: Assume \( m \geq 1 \) and \( \| x_k - v_i \|_A > 0, 1 \leq i \leq c, 1 \leq k \leq n, 1 \leq p \leq S \). \((U, V)\) may minimize \( J_m \) only if:

\[
\mu_{ik} = \left[ \sum_{j=1}^{c} \left( \frac{\| x_k - v_j \|_A}{\| x_k - v_j \|_A} \right)^{\frac{m}{m-1}} \right]^{-1} 
\]

\[
u_{ip} = \frac{\sum_{k=1}^{n} (\mu_{ik})^m x_k}{\sum_{k=1}^{n} (\mu_{ik})^m} 
\]

The FCM algorithm consists of iterations alternating between equations (5) and (6). This algorithm converges to either a local minimum or a saddle point of \( J_m \)[11]. The codebook vector \( v_i \) defined in (6) is the Euclidean center of gravity or centroid of all the training vectors assigned to the \( i \)th cluster. When applied to image histogram segmentation, a Fast version of FCM (FFCM) [12] is preferred to conventional one [11] due to its computational cost. The proposed algorithm [12] is based on one dimensional attribute such as the gray-level. Let \( H \) be the histogram of image of \( L \)-levels, where \( L \) is the number of gray levels. Each pixel has a feature that lies in the discrete set \( X = \{0, 1, \ldots, L - 1\} \).

In the new formulation [12], the FFCM minimizes the following functional, which is very similar to that of [11]:

\[
\Phi_m(U, G : L) = \sum_{l=0}^{L-1} \sum_{i=0}^{c} (\mu_{il})^m H(l). (l - g_i)^2 
\]

where \( g_i \) is the centroid of the cluster \( i \) (\( i \)th codebook). The derivation of the FFCM is based on the constrained minimization of the objectif function (7). \((U, G)\) may minimize \( \Phi_m(U, G) \) only if:

\[
\tilde{\mu}_{il} = \left[ \sum_{l=0}^{L-1} \frac{1 - g_i}{|l - g_i|} \right]^{\frac{1}{m-1}} 
\]

\[
g_i = \frac{\sum_{l=0}^{L-1} (\mu_{il})^m H(l). l}{\sum_{l=0}^{L-1} (\mu_{il})^m H(l)} 
\]

The FFCM only operates on the histogram and hence is faster than the conventional version which processes the whole data set. Thus, the computation of the membership degrees of \( H(l) \) pixels is reduced to that of only one pixel with \( l \) as gray level value.

3. Image compression method

To construct the VQ codebook the FFCM algorithm is used to initialize the codebook followed by a fine construction using the FCM algorithm. The compression method proceeds as follows:

1. Fix the size \( c \) of the codebook, \( 2 \leq c \leq L \) and the threshold value \( \epsilon \).

2. Find the number of occurrences \( H(l) \) of the level \( l, 0 \leq l \leq L - 1 \).

3. Initialize randomly the membership degree \( \mu_{il} \) using the \( L \) gray levels such that \( \sum_{i=1}^{c} \mu_{il} = 1 \).

4. Compute the codeword \( g_i \) using equation (9).

5. Update the membership degrees \( \tilde{\mu}_{il} \) using equation (8).

6. Compute the defect measure \( E = \sum_{l=0}^{L-1} \sum_{i=0}^{c} | \tilde{\mu}_{il} - \mu_{il} | \).

7. If \( (E > \epsilon) \) \( \mu_{il} \leftarrow \tilde{\mu}_{il} \) and goto Step 4.

8. Defuzzification process.

9. Initialize the codeword: \( v_{ip} \leftarrow g_i \)

10. Repeat Steps 4 through 7 using equation (5) and (6).

11. Defuzzification process.

12. Decoding: \( x_{kp} \leftarrow v_{ip} \) (using maximum membership degree rule, the \( k \)th training vector is affected to the \( i \)th cluster).

4. Results

The performance of the proposed method is evaluated on PC Pentium II. The training vectors used for codebook design are obtained from 256 x 256 standard test image. The proposed compression method has been successfully applied to a number of tested images. Only the result of one image (Lenna image) is presented. The length of the codebook has been varied from \( 2^4 \) to \( 2^8 \) and the sizes of blocks from \( 2 \times 2 \) to \( 8 \times 8 \).
to 4 × 4. Each block of pixels has been converted into a vector by scanning the rows of the block. As commonly used measure in a variety of coding systems, Peak Signal-to-Noise Ratio (PSNR) is adopted for evaluating objective quality. PSNR is defined as

$$\text{PSNR} = 10 \log_{10} \frac{E[||X||^2]}{E[d(X, \hat{X})]}$$

(10)

where \(E[.]\) is the mathematical expectation and \(d(X, \hat{X})\) is the distortion measure that is assigned to a cost of reproduction of a value in a codebook \(\hat{X}\) according to an input value \(X\). Compression is obtained in VQ by using a codebook with relatively few codevectors compared to the number of possible image vectors. The resulting bit rate of a VQ scheme is

$$R = \log_2(N_c)/S$$

(11)

where \(N_c\) is the codebook size and \(S\) the number pixels of the block. The compression ratios along with PSNR for reconstructed images with two block sizes (2 × 2, 4 × 4) and varying codebook length, from 16 to 64 are given in Table 1. The analysis of the obtained results shows a significant increase in the compression ratio when the size of the block is increased from 2 × 2 pixels per block to 4 × 4 pixels per block. This results in a reduction in PSNR of the reconstructed images. However, increase in the compression ratio may be achieved by reduction of the codebook length (Table 1). The reduction in the length of the used codebook from 64 to 16 gives relatively small loss in PSNR. Note that the blocking effect is more apparent for 4 × 4 (Figs. 1(b)-(d)) than for 2 × 2 (Figs. 1(e)-(g)). The final choice of compression ratio is greatly depends on the required visual quality. Although still widely used, the PSNR generally does not correlate well with subjective judgments. Thus, development of objective quality measure for image compression based on the human visual system is urgently required.

References


