ADAPTIVE LIFTING SCHEMES WITH A GLOBAL ℓ_1 MINIMIZATION TECHNIQUE FOR IMAGE CODING

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ABSTRACT

Many existing works related to lossy-to-lossless image compression are based on the lifting concept. In this paper, we present a sparse optimization technique based on recent convex algorithms and applied to the prediction filters of a two-dimensional non separable lifting structure. The idea consists of designing these filters, at each resolution level, by minimizing the sum of the ℓ_1-norm of the three detail subbands. Extending this optimization method in order to perform a global minimization over all resolution levels leads to a new optimization criterion taking into account linear dependencies between the generated coefficients. Simulations carried out on still images show the benefits which can be drawn from the proposed optimization techniques.

Index Terms— adaptive lifting scheme, image coding, optimization, ℓ_1 minimization techniques, sparse representations.

1. INTRODUCTION

The discrete wavelet transform has been recognized to be an efficient tool in many image processing fields, including denoising and compression [1]. In this respect, the second generation of wavelets provides very efficient transforms, based on the concept of Lifting Scheme (LS) developed by Sweldens [2]. It was shown that interesting properties are offered by such structures. In particular, LS guarantees a lossy-to-lossless reconstruction required in some specific applications such as medical imaging or remote sensing imaging [3]. Besides, it is a suitable tool for scalable reconstruction, which is a key issue for telebrowsing applications [4, 5].
A generic LS applied to a 1D signal consists of three modules referred to as split, predict and update. Generally, for 2D signals, the LS is handled in a separable way by cascading vertical and horizontal 1D filtering operators. It is worth noting that a separable LS may not appear always very efficient to cope with the two-dimensional characteristics of edges which are neither horizontal nor vertical [6]. To this respect, several research works have been devoted to the design of Non Separable Lifting Schemes (NSLS) in order to offer more flexibility in the design of the prediction filter [7, 8, 9, 10]. Thus, instead of using samples from the same rows (resp. columns) while processing the image along the lines (resp. columns), 2D NSLS provide more choices in the selection of the samples by using horizontal, vertical and oblique directions. Moreover, in a coding framework, the performance of these LS depends on the choice of the prediction and update operators. For this reason, a great attention was paid to the *optimization* of all the involved filters in order to build *content*-adaptive schemes. Only a few works have discussed the problem for the update filter. The state-of-the-art method consists of designing the update operator so that the reconstruction error is minimized when the detail coefficients are canceled [11, 12]. Recently, we have designed an update filter that aims at reducing the aliasing effects [13]. It was designed by minimizing the difference between its output and the output of an ideal low-pass filter. Besides, most existing works have been focused on the optimization of the prediction filters. In [8], these filters are designed by minimizing the entropy of the detail coefficients. This optimization is performed in an empirical manner using the Nelder-Mead simplex algorithm since the entropy is an implicit function of the prediction filter. However, such a heuristic algorithm presents two major drawbacks. First, its convergence may be achieved at a local minimum of entropy. Second, it is computationally intensive. To overcome these problems, a simpler criterion, measuring the variance of the detail signal (i.e. its ℓ_2-norm), has been often used to optimize the prediction filters [12, 14]. With the ultimate goal of promoting sparsity in a transform domain, we have recently proposed sparse optimization criteria for the design of these filters. We have focused on the use of ℓ_1 and weighted ℓ_1 criteria [15]. It is worth pointing out here that the aforementioned optimizations are generally performed by applying the criterion on the *current* resolution level of the decomposition. In this paper, we propose a significant improvement of our recent optimization technique by minimizing a criterion evaluated over *all* the resolution levels and by employing a Douglas-Rachford algorithm in a product space [16]. This method is shown to provide better coding performance in terms of quality of reconstruction (up to 0.35 dB) and bitrate savings (up to -11%).
The outline of the paper is as follows. In Sec. 2, we introduce the sparse optimization problem for the design of the prediction filters involved in a 2D non separable lifting structure. The global optimization strategy is described in Sec. 3. Finally, in Sec. 4, experimental results are given and some conclusions are drawn in Sec. 5.

2. ADAPTIVE LIFTING STRUCTURE

2.1. 2D Non-separable lifting structure

In this paper, we consider a 2D NSLS composed of three prediction lifting steps followed by an update lifting step (see Fig. 1 for the analysis part). The interest of this structure is twofold. First, it allows us to reduce the number of lifting steps and rounding operations. A theoretical analysis has been conducted in [17] showing that NSLS improves the coding performance due to the reduction of rounding effects. Furthermore, any separable prediction-update (P-U) LS structure has its equivalent in this form [10, 17].

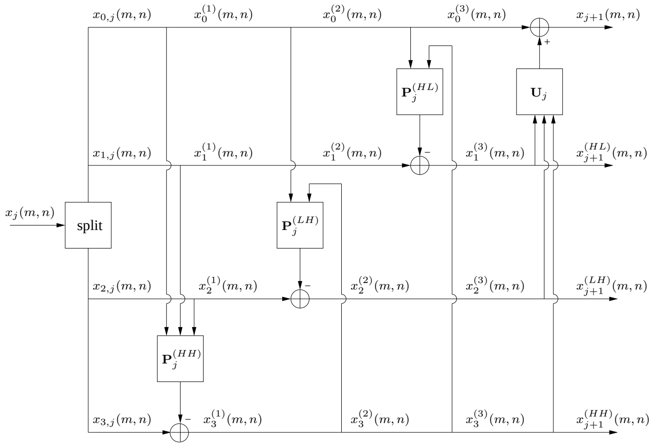

Fig. 1. NSLS decomposition structure.

Let x denote the input image to be coded. At each resolution level j and each pixel location (m,n), the approximation coefficient is denoted by $x_j(m,n)$ and the associated four polyphase components by $x_{0,j}(m,n) = x_j(2m,2n)$, $x_{1,j}(m,n) = x_j(2m,2n+1)$, $x_{2,j}(m,n) = x_j(2m+1,2n)$, and $x_{3,j}(m,n) = x_j(2m+1,2n+1)$. Furthermore, we denote by $\mathbf{P}_j^{(HH)}$, $\mathbf{P}_j^{(LH)}$, $\mathbf{P}_j^{(HL)}$ and $\mathbf{U}_j$ the three prediction and the update filters employed to generate the detail coefficients $x_{j+1}^{(HH)}$ oriented diagonally, $x_{j+1}^{(LH)}$ oriented vertically, $x_{j+1}^{(HL)}$ oriented horizontally, and the approximation coefficients x_{j+1}. Once the considered NSLS structure is defined, we now focus on the optimization of its lifting operators.

2.2. ℓ_1 and weighted ℓ_1 minimization technique

Based on the requirement of sparse detail coefficients, we propose the use of an ℓ_1-based criterion in place of the usual ℓ_2-based measure (variance of the prediction error). For this reason, we have firstly optimized the prediction filters by minimizing at each resolution level j the following ℓ_1 criterion:

$$\forall\, o \in \{HL, LH, HH\}, \forall\, i \in \{1,2,3\},$$

$$\widetilde{\mathcal{J}}_{j,\ell_1}^{(o)}(\mathbf{p}_j^{(o)}) = \sum_{m=1}^{M_j}\sum_{n=1}^{N_j} \left| x_{i,j}(m,n) - (\mathbf{p}_j^{(o)})^\top \mathbf{X}_j^{(o)}(m,n)\right| \quad (1)$$

where $x_{i,j}(m,n)$ is the $(i+1)^{th}$ polyphase component to be predicted, $\mathbf{X}_j^{(o)}(m,n)$ is the reference vector containing the samples used in the prediction step, $\mathbf{p}_j^{(o)}$ is the prediction operator vector to be optimized, M_j and N_j corresponds to the dimensions of the input subband x_{j+1}. Although the criterion in (1) is convex, a major difficulty that arises in solving this problem stems from the fact that the function to be minimized is not differentiable. For this reason, we resort to the class of proximal optimization algorithms which have been proposed to solve nonsmooth minimization problems like (1) [16]. More precisely, we employ the Douglas-Rachford algorithm which is an efficient optimization tool in this context [18].
Moreover, it can be noticed from Fig. 1 that the diagonal detail signal $x_{j+1}^{(HH)}$ is also used through the second and the third prediction steps to compute the vertical and the horizontal detail signals respectively. As a result, it would be interesting to optimize the prediction filter $\mathbf{p}_j^{(HH)}$ by minimizing the following *weighted* sum of the ℓ_1-norm of the three detail subbands $x_{j+1}^{(o)}$:

$$\widetilde{\mathcal{J}}_{j,w\ell_1}(\mathbf{p}_j^{(HH)}) = \sum_{o\in\{HL,LH,HH\}} \sum_{m=1}^{M_j}\sum_{n=1}^{N_j} \frac{1}{\alpha_{j+1}^{(o)}} \left| x_{j+1}^{(o)}(m,n)\right| \quad (2)$$

where $\alpha_{j+1}^{(o)}$ can be estimated by using a classical maximum likelihood estimate [19]. It can be noticed that (2) is related to the approximation of the entropy of an i.i.d Laplacian source. After expressing for each orientation $o \in \{HH, HL, LH\}$ the signal $x_{j+1}^{(o)}$ as a function of the filter $\mathbf{p}_j^{(HH)}$ (by assuming that $\mathbf{p}_j^{(LH)}$ and $\mathbf{p}_j^{(HL)}$ are known), we can also use the Douglas-Rachford algorithm, reformulated in a *three-fold product space* [16], to minimize the proposed weighted criterion (2). It is important to note here that the optimization of the filter $\mathbf{p}_j^{(HH)}$ depends on the coefficients of the filters $\mathbf{p}_j^{(HL)}$ and $\mathbf{p}_j^{(LH)}$ since the weighted sum of the ℓ_1-norm of the three detail subbands is minimized. On the other hand, the optimization of the filters $\mathbf{p}_j^{(HL)}$ and $\mathbf{p}_j^{(LH)}$ depends also on the optimization of the filter $\mathbf{p}_j^{(HH)}$ since $x_{j+1}^{(HH)}$ is used as a reference signal in the second and the third prediction steps. For this reason, a *joint* optimization method needs to be used, which alternates between optimizing the different filters and redefining the weights. More precisely, the main idea of this method starts with an initialization step where each prediction filter $\mathbf{p}_j^{(o)}$ is separately optimized by minimizing $\widetilde{\mathcal{J}}_{\ell_1}(\mathbf{p}_j^{(o)})$, and the resulting weighting terms are then computed. After that, we iteratively repeat the following two steps: reoptimize the filters $\mathbf{p}_j^{(HH)}$, $\mathbf{p}_j^{(LH)}$ and $\mathbf{p}_j^{(HL)}$ by minimizing respectively $\widetilde{\mathcal{J}}_{j,w\ell_1}(\mathbf{p}_j^{(HH)})$, $\widetilde{\mathcal{J}}_{j,\ell_1}^{(LH)}(\mathbf{p}_j^{(LH)})$ and $\widetilde{\mathcal{J}}_{j,\ell_1}^{(HL)}(\mathbf{p}_j^{(HL)})$, and update the weighting terms. Note that the convergence of the proposed joint optimization algorithm is achieved during the early iterations (after about 7 iterations) [15].

3. GLOBAL MULTIRESOLUTION ℓ_1 MINIMIZATION

So far, we have optimized the prediction filters $\mathbf{p}_j^{(o)}$ by minimizing a criterion related directly to their respective outputs $x_{j+1}^{(o)}$. As previously mentioned, in other approaches, the lifting operators are also designed at each resolution level by optimizing a criterion defined at the current level. However, such an optimization procedure presents the following drawback: in a multiresolution representation where the decomposition structure given by Fig. 1 is applied iteratively on the approximation coefficients, it can be noticed that the detail coefficients $x_{j+1}^{(o)}(m,n)$, resulting from the optimization of the filters $\mathbf{p}_j^{(o)}$ are also used to compute the detail coefficients at the coarser resolution levels. Thus, the optimization of the coefficients at the j-th resolution level will also affect the detail coefficients at the $(j+1)$-th level. Due to this fact, the solution $\mathbf{p}_j^{(o)}$ resulting from the previous optimization method may be suboptimal. To circumvent this problem, we propose to optimize each prediction filter by minimizing a *global* criterion computed over *all* the resolution levels. More precisely, instead of minimizing the sum of the ℓ_1-norm of the three detail subbands (see Eq. (2)) , we will consider the minimization of the following criterion:

$$\mathcal{J}_{w\ell_1}(\mathbf{p}) = \sum_{j'=1}^{J} \sum_{o'\in\{HL,LH,HH\}} \sum_{m,n} \frac{1}{\alpha_{j'}^{(o')}} \left| x_{j'}^{(o')}(m,n)\right| \quad (3)$$

where J corresponds to the number of resolution levels and $\mathbf{p} = \left(\mathbf{p}_j^{(o)}\right)_{j,o}$. Hence, the proposed minimization problem leads to a joint optimization procedure for the prediction filters $\mathbf{p}_j^{(o)}$ (i.e. $3J$ unknowns). This problem is addressed by an alternating optimization approach. In order to apply the proposed optimization method, we express the detail signal $x_{j'}^{(o')}(m,n)$ in Eq. (3) as a function of the filter $\mathbf{p}_j^{(o)}$ to be optimized. To this end, the filters at each resolution level j and orientation o will be re-indexed by $q \in \{1,\ldots,3J\}$. According to Fig. 1, let $\left(x_i^{(q)}(m,n)\right)_{i\in\{0,1,2,3\}}$ be the four outputs obtained from the inputs $\left(\tilde{x}_i^{(q-1)}(m,n)\right)_{i\in\{0,1,2,3\}}$ at the q-th prediction step:

$$\tilde{x}_i^{(q-1)}(m,n) = \begin{cases} x_{i,j}(m,n), & \text{if } q = 3j+1 \\ x_i^{(q-1)}(m,n), & \text{otherwise.} \end{cases} \quad (4)$$

Then, it can be shown that:

$$\begin{aligned} &\forall\, o' \in \{HH, LH, HL\},\ \forall\, j' > j \\ &x_{j'}^{(o')}(m,n) = y_{j'}^{(o',q)}(m,n) - (\mathbf{p}_j^{(o)})^\top \mathbf{x}_{j'}^{(o',q)}(m,n) \end{aligned} \quad (5)$$

where

$$\begin{aligned} y_{j'}^{(o',q)}(m,n) = &\sum_{i'\in\mathbb{I}_i}\sum_{k,l} h_{i',j'}^{(o',q)}(k,l)\, x_{i'}^{(q)}(m-k,n-l) \\ &+ \sum_{k,l} h_{i,j'}^{(o',q)}(k,l)\, \tilde{x}_i^{(q-1)}(m-k,n-l), \end{aligned} \quad (6)$$

$$\mathbf{x}_{j'}^{(o',q)}(m,n) = \Big(\sum_{k,l} h_{i,j'}^{(o',q)}(k,l)\, \tilde{x}_{i'}^{(q-1)}(m-k-r,n-l-s)\Big)_{\substack{(r,s)\in\mathcal{P}_j^{(o)}\\ i'\in\mathbb{I}_i}} \quad (7)$$

with $i = 3(j+1)+1-q$ and $\mathbb{I}_i = \{0,1,2,3\}\backslash\{i\}$.

It is important to note that, in practice, the computation of $y_{j'}^{(o',q)}(m,n)$ and $\mathbf{x}_{j'}^{(o',q)}(m,n)$ for the q-th prediction step does not require to find the explicit expressions of the impulse response $h_{i,j'}^{(o',q)}$ since these signals can be determined numerically as follows:

- The first term (resp. the second one) in the expression of $y_{j'}^{(o',q)}(m,n)$ in Eq. (6) can be found by computing $x_{j'}^{(o')}(m,n)$ from the components $\left(x_{i'}^{(q)}(m,n)\right)_{i'\in\mathbb{I}_i}$ while setting $x_i^{(q)}(m,n) = 0$ (resp. while setting $x_{i'}^{(q)}(m,n) = 0$ for $i' \in \mathbb{I}_i$ and $x_i^{(q)}(m,n) = \tilde{x}_i^{(q-1)}(m,n)$).
- The vector $\mathbf{x}_{j'}^{(o',q)}(m,n)$ in Eq. (7) can be found as follows. For each $i' \in \mathbb{I}_i$, the computation of $\sum_{k,l} h_{i,j'}^{(o',q)}(k,l)\tilde{x}_{i'}^{(q-1)}(m-k,n-l)$ requires to compute $x_{j'}^{(o')}(m,n)$ by setting $x_i^{(q)}(m,n) = \tilde{x}_{i'}^{(q-1)}(m,n)$ and $x_{i''}^{(q)}(m,n) = 0$ for $i'' \in \mathbb{I}_i$. The result of this operation has to be considered for different shift values (r,s) (as can be seen in Eq. (7)). Once the different terms have been defined, one can still employ the Douglas-Rachford algorithm in a product space to minimize the proposed criterion.

4. EXPERIMENTAL RESULTS

Since we are interested in the optimization of the different filters involved in a NSLS (which is equivalent to the 2D structure of any P-U LS), we will consider the 5/3 transform also known as the (2,2) wavelet transform. In what follows, this method will be designated by "NSLS(2,2)". In order to show the benefits of the proposed optimization methods, we provide the results for the following decompositions carried out over three resolution levels. The first one corresponds to the state-of-the-art prediction optimization method based on the minimization of the ℓ_2 norm of the detail coefficients [12, 14]. This method will be denoted by "NSLS(2,2)-OPT-L2". In the second method, we *jointly* optimize the prediction filters by using the weighted ℓ_1 minimization technique (see Eqs. (1)-(2)). In the following, this method will be designated by "NSLS(2,2)-OPT-WL1". Finally, the proposed extension of this method, where a global weighted ℓ_1 criterion is defined over *all* the resolution levels (see Eq. (3)), will be denoted by "NSLS(2,2)-GLOBAL-OPT-WL1". Fig. 3 displays the scalability in quality of the reconstruction procedure by providing the variations of the PSNR versus the bitrate for the "peppers" image using JPEG2000 as an entropy codec. These plots show that promoting sparsity criteria by using a weighted ℓ_1 criterion ("NSLS(2,2)-OPT-WL1") achieves a gain of about 0.25 dB compared to the conventional ℓ_2 minimization technique ("NSLS(2,2)-OPT-L2"). Furthermore, an improvement of about 0.3 dB is further obtained by using the proposed *global* weighted ℓ_1 minimization approach. Fig. 2 displays a zoom applied on the reconstructed images at 0.07 bpp. The quality of these images is evaluated in terms of PSNR and SSIM metrics. Finally, in order to measure the relative gain of the proposed optimization method, we used the Bjontegaard metric [20]. The results are shown in Tables 1 and 2 for low bitrate, middle bitrate and high bitrate corresponding respectively to the four bitrate points $\{0.15, 0.2, 0.25, 0.3\}$, $\{0.5, 0.55, 0.6, 0.65\}$ and $\{0.9, 0.95, 1, 1.05\}$ bpp. Table 1 (resp. 2) gives the gain of the method "NSLS(2,2)-OPT-WL1" (resp. "NSLS(2,2)-GLOBAL-OPT-WL1") compared with 'NSLS(2,2)-OPT-L2". Note that a bitrate saving with respect to the reference method corresponds to negative values. It can be observed that the proposed *global* minimization approach can outperform the classical ℓ_2 (resp. weighted ℓ_1) minimization technique up to -16% and 0.6 dB (resp. -11% and 0.35 dB) in terms of bitrate saving and quality of reconstruction. Moreover, we should note that, for some images, the global weighted ℓ_1 criteria resulting from the minimization over all the resolution levels and from the minimization at each resolution level take close values. In these cases, we obtain a slight improvement in terms of bitrate.

5. CONCLUSION

In this paper, we have presented different optimization methods for the design of the prediction filters in a non separable lifting structure. A sparse optimization technique, involving the minimization of a global criterion defined over all the resolution levels, has been proposed. Experiments have shown the benefits of the proposed method. Ongoing research aims at extending this optimization approach to LS with more than two stages like the P-U-P and P-U-P-U structures.

Table 1. The average PSNR differences and the bitrate saving. The gain of "NSLS(2,2)-OPT-WL1" w.r.t "NSLS(2,2)-OPT-L2".

	bitrate saving (in %)			**PSNR gain (in dB)**		
Images	**low**	**middle**	**high**	**low**	**middle**	**high**
Elaine	-6.40	-5.33	-3.29	0.14	0.12	0.15
Castle	-7.10	-7.15	-6.09	0.30	0.48	0.62
Straw	-1.80	-5.03	-3.33	0.08	0.23	0.19
Peppers	-6.72	-5.53	-8.64	0.25	0.19	0.36
Average	-5.50	-5.76	-5.34	0.19	0.25	0.33

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(a) Original image (b) PSNR=27.19 dB, SSIM=0.752 (c) PSNR=27.86 dB, SSIM=0.761

Fig. 2. Reconstructed "peppers" image at 0.07 bpp using (b) NSLS(2,2)-OPT-L2 (c) NSLS(2,2)-GLOBAL-OPT-WL1.

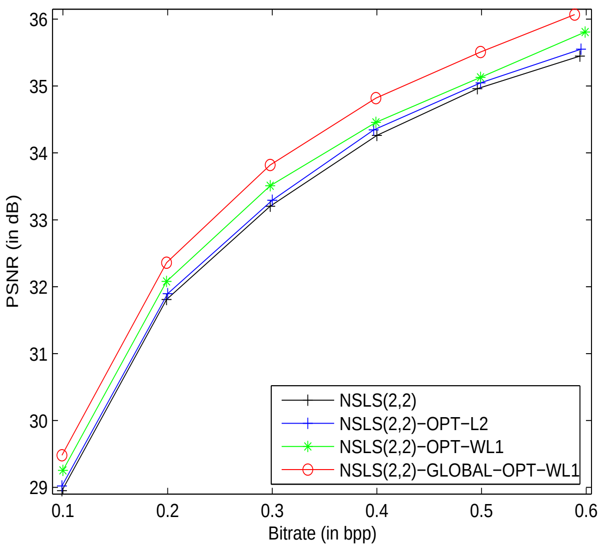

Fig. 3. PSNR (in dB) versus the bitrate (in bpp) after JPEG2000 progressive encoding of 'peppers' image.

Table 2. The average PSNR differences and bitrate saving. The gain of "NSLS(2,2)-GLOBAL-OPT-WL1" w.r.t "NSLS(2,2)-OPT-L2"

	bitrate saving (in %)			**PSNR gain (in dB)**		
Images	**low**	**middle**	**high**	**low**	**middle**	**high**
Elaine	-7.99	-11.56	-10.26	0.16	0.29	0.45
Castle	-7.57	-8.13	-6.45	0.32	0.54	0.66
Straw	-2.80	-6.37	-3.89	0.10	0.29	0.23
Peppers	-13.73	-16.43	-15.76	0.54	0.54	0.67
Average	-8.02	-10.62	-9.09	0.28	0.41	0.50

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