A New Adaptive Nonlinear Anisotropic Diffusion for Noise Smoothing.

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Abstract

The aim of this work is to propose an adaptive nonlinear filtering method based on the nonlinear anisotropic diffusion equation. This new method has the advantage of flexibility and adaptability. Furthermore, A new degradation effect, not signaled before, we call «pinhole effect», which may result in anisotropic diffusion is introduced and analyzed. A robust solution to this effect is proposed and evaluated through experimental data. The performance of the method is demonstrated on actual and synthetic images.

1. Introduction

Since the pioneer work of Kavasznay and Joseph [1] , Partial Differential Equations (PDE's) are increasingly used in digital image treatment as an efficient approach for image enhancement. A special issue of the IEEE Transactions on Image Processing Journal has been devoted to this subject this year. It has been proved through a great number of works that nonlinear filters overcome their linear counterpart in smoothing out noise without sensibly affecting the salient futures of the signal. Nonlinear filters could be classified into four main groups : the first concerns order statistics filters [2] which are based on the order statistics, the second group corresponds to morphological filters developped by Serra at Ecole des Mines de Paris [3], the third are based on PDE's [4] and the last group is less known and it concerns the methods based on the Human Visual System [5-6]. The present work, based on the PDE which governs a nonlinear anisotropic diffusion in a medium, belongs to the third group. The first paper which clearly gave a detailed analysis of the nonlinear anisotropic diffusion and its digital implementation is that of Perona and Malik [7]. The basic idea of the well known Perona and Malik's method is to consider the image signal as a medium where diffusion can take place with a variable conductance in order to control the smoothing effect. One way to achieve this goal is to allow the conductance function to vary according to the local gradient of the image intensity. Thus, by choosing a decreasing function of the gradient one can adaptively control the amount of smoothing effect. Indeed, this controlled diffusion results in less smoothing in regions with relevant image features, and more smoothing in noisy homogeneous regions. In contrast with linear filters, this process has twofold : noise smoothing and edge enhancement. The system of equations governing this phenomenon has an initial condition which is, for instance, the original degraded image. When the diffusion process is iterated a set of smoothed versions of the original image is obtained. However, some drawbacks and limitations of this original model have been mentioned in the literature [8-10]. For instance, the conductance function was chosen in a heuristic manner in the Perona and Malik's method. Catté et al. [8] have proved the ill-posedness, in some cases, of the diffusion equation when using the conductance functions proposed by Perona and Malik. Whitaker and Pitzer have pointed out the staircasing effect that may occur on wide smooth edges [9]. Besides these works, an interesting strategy has been recently proposed by Li and Chen [10]. The idea of Li and Chen is to use a decreasing function of the gradient such that used in [7] and to adapt the conductance parameter κ during the diffusion process evolution. The aim of this work is to propose an adaptive nonlinear filtering method based on the anisotropic diffusion which avoids some drawbacks of similar existing techniques. A judicious choice of the conductance function and an automatic strategy for computing the edgeness threshold are proposed and evaluted. A more detailed and elaborated analysis of these proposed improvements is given in [11].

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2. Standard Nonlinear Anisotropic Diffusion and some improvements.

Before introducing our improvements, let us recall Perona and Malik's method which we call Standard Nonlinear Anisotropic Diffusion (SNAD). The image is considered as a medium where a fluid can diffuse in an anisotropic manner. Given an image at its initial state, say at $t = 0$, the spatio-temporal evolution of the image intensity $I(\mathbf{X},t)$ is described by the heat equation given by :

$$\frac{\partial I}{\partial t} = \nabla^T \left(c(\mathbf{X},t)\nabla I(\mathbf{X},t)\right) \quad (1)$$

where, $c(\mathbf{X},t)$ is the conductance function. The aim of using this equation is to control the amount of smoothing according to the local signal features : less smoothing across edges and more smoothing along edges and inside homogeneous regions. Note that when these two effects are combined, object/background transitions are enhanced and the noise and weak image features are smoothed out in the same time. One way to achieve this goal is to define a conductance function which varies with the local gradient amplitude : $c(\|\nabla I(\mathbf{X},t)\|)$. The idea of Perona and Malik is to chose a decreasing conductance function of the local gradient magnitude in order to insure high diffusion in homogeneous regions and along edges but null, or at least weak, diffusion across edges. Perona and Malik suggested two heuristic functions satisfying these requirements.

$$c_1(\mathbf{X},t) = \exp\left(-\frac{\|\nabla I(\mathbf{X},t)\|}{\kappa}\right)^2 \qquad (2)$$

In this function κ is a parameter which controls the strength of the diffusion. Catté et al. [8] have shown that for some gradient values the problem turns to an inverse heat equation which is known to be an ill-posed problem. One way to overcome this difficulty is to introduce another scaling parameter σ intervening in an implicit gaussian filtering in order to better estimate the contour gradients and the edgeness threshold. Whitaker and Pizer suggested to chose a decreasing function of time σ(t). But they do not give a strategy in order to automatically adapt this parameter to the local properties of the image signal. Moreover, these authors have noticed that an undesirable degradation, they called the « staircasing effect », could raise in wide smooth regions. Li and Chen [10] used the same idea of Whitaker and Pizer but adopted another strategy, where the conductance parameter κ is adapted to the signal activity during the diffusion process evolution. Li and Chen suggest to chose κ as a decreasing function during the process evolution. Nevertheless, this function is empirically chosen and κ is not signal dependant.

The aim of the proposed study is to give a strategy which may help in selecting the value of κ in an automatic and adaptive fashion as the diffusion process evolves.

3. Adaptive Nonlinear Anisotropic Diffusion

For the sake of clarity our approach called Adaptive Nonlinear Anisotropic Diffusion (ANAD) will be presented by analyzing step by step the improvements we have made. The first analyzed parameter is the conductance function, the second point to be discussed is the pin-hole effect and the third one is the edgeness threshold computation strategy we have adopted.

3.1. Analysis of the conductance function behavior

Perona and Malik suggest two types of conductance functions that share the same property of decreasing with the local gray-level gradient. However, Perona and Malik did not give a strategy to control these functions during the diffusion evolution. In the present study we use a more bell-behaved conductance function which shares the property of preserving the transition range whatever the κ value . An example of such function is for example :

$$c_2(x,t) = 0.5\,[\tanh\left(\gamma\left(\kappa - \|\nabla I(x,t)\|\right)\right) + 1] \quad (3)$$

The constant γ controls the steepness of the min-max transition region whereas κ controls the extent of the diffusion region in term of gradient gray-level. In all the experiments the γ parameter is fixed to 0.02. This value is experimentally proved to be adequate for a great variety of actual images.

3.2. Pinhole effect

A new phenomenon, not signaled before, and which could occur in diffusion process is addressed in the following. It concerns what we call « pinhole effect ». It happens when a pixel or a group of adjacent pixels with a gray-level of intermediate value is near a sharp transition region. When diffusion takes place, this point (called « vanishing point ») serves as a junction between the two regions leading to an undesirable smoothing effect through edges. In order to avoid this undesirable effect, a simple method is proposed and evaluated in the following. To localize these points we examine the gradient of the current pixel and those of its neighbors. Let μ_-, μ_+ and μ_0 represent, respectively, the mean gray-level of pixels with negative gradient, that of pixels with positive gradient and that of the central pixel. For a given gradient threshold value κ_s, the central pixel is considered as a vanishing point if the following conditions are simultaneously satisfied :

$$
\begin{aligned}
&0 \le \mu_0 - \mu_- \le \kappa_s, \\
&0 \le \mu_+ - \mu_0 \le \kappa_s \qquad (4) \\
&\kappa_s \le \mu_+ - \mu_-
\end{aligned}
$$

The method of computing the edgeness threshold κ_s will be addressed later. When such point is detected the diffusion is stopped to avoid the pinhole effect or better one can remove it using any nonlinear filter such as morphological filters.

3.3. Edgeness threshold estimation strategy

Let us now describe the method of computing the critical parameter κ_s , called the edgeness threshold , which allows to control the amount of smoothing effect. The image is subdivided into non-overlapping blocks B_k of the same size, typically between 4x4 and 32x32 pixels. A local homogeneity or uniformity measure is then defined and measured in each block. Since, there is no universal definition one can use any measure where local gray-level variations can be captured. Once this measure is computed the blocks are sorted in homogeneity decreasing way. Only a fraction of the blocks with high homogeneity measure are retained. Since the noise is assumed stationary and randomly distributed in the image space, then a practical way to estimate its variance is to consider homogeneous regions where small variations or texture are mainly due to noise. Then, we compute the histogram of gray-level gradient in these selected blocks and use the maximum of this histogram as a good indicator for the gradient threshold to use in our diffusion scheme, i.e. κ_s is set to this value. This analysis is performed at each iteration and a new threshold κ_s value is automatically determined. As mentioned before, the conductance function c_2 could be easily controlled by the κ parameter which fixes the gradient threshold. This critical value defines two different behaviors of the diffusion : blurring effect if $\|\nabla I\| \le \kappa_s$ and edge sharpening for $\|\nabla I\| > \kappa_s$.

4. Experimental evaluation of the method

In order to evaluate the efficiency of the method, images corrupted by an additive gaussian white noise are considered. The first test image (fig.1a) is an actual noisy text image where the pin-hole effect is simulated by noisy pixels in the vicinity of object/background transitions. Figures 1b and 1c show the obtained results when using the method SNAD of Perona and Malik with two different values of κ_s and after 6 iterations. When pin-hole effect is considered and treated, the results shown on fig.1d is obtained. To point out the pin-hole effect, which may result when using SNAD method, a selected zone is zoomed and displayed. Figure 1c shows the results obtained when using SNAD method and fig.1d illustrates the pin-hole effect correction.

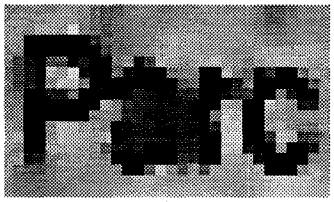
(a) Original noisy image.

(b) SNAD with c_2, κ_s = 50 , 6 iterations.

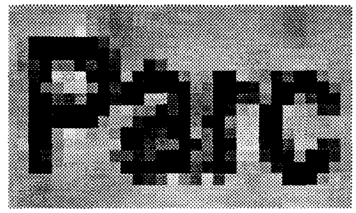
(c) SNAD with c_2, κ_s =100, 6 iterations

(d) Proposed method with c_2, κ_s =100, 6 iterations

Figure 1. (Continued)



(e) Zoomed region of image1c



(f) Zoomed region of image (1d)

Figure 1. Pine-hole effect in actual image - comparison of classical SNAD with SNAD with pin-hole effect correction.

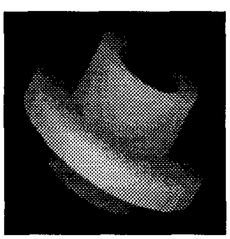
Now, the proposed method will be compared with two other known methods more. In order to better evaluate and follow the smoothing effect of the considered methods a synthetic image is considered. This test image (fig.2a) contains many transition regions with various orientations and contrast . Figure 2b shows the results of corrupting this image by a gaussian white noise. The results of applying SNAD method are shown in fig.2c. The following figures, fig2.d through fig.2f, illustrate the obtained results when using the method of Whitaker and Pizer (WP), that of Li and Chen (LC) and our proposed method, respectively.

For subjective comparison only visual judgment is used whereas for objective comparison we use the Normalize Mean Square Error (NMSE) given by :

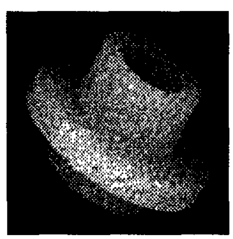
$$NMSE = \sum_{i,j}[Y(i,j)-S(i,j)]^2 \Big/ \sum_{i,j}[X(i,j)-S(i,j)]^2 \quad (5)$$

where S(i,j), X(i,j), and Y(i,j) are the original, noisy input, and filtered images, respectively.

The main objective of this comparison is to demonstrate the flexibility and adaptability of ANAD method. Indeed, in contrast with the other cited algorithms, the proposed method does not require tuning the κ parameter. In our filtering technique this parameter is signal dependant and it is adaptively and automatically adapted to the signal/noise characteristics. Although, it could be noticed that, in the basis of the NMSE and subjective visual criterion, the proposed method gives better results than the other methods.

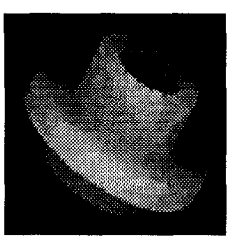


(a) Original image

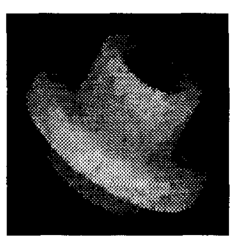


(b) Noisy image

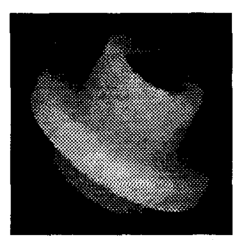
(c) SNAD method
NMSE = 0,1945 (5 trials)



(d) WP method
NMSE = 0,2185 (5 trials)



(e) LC method
NMSE = 0,2212 (5 trials)



(f) ANAD method
NMSE = 0,1932 (1 trial)

Figure 2. Comparison of ANAD with SNAD, WP and LC methods.

The superiority of the proposed method over the others is essentially its adaptability and flexibility. Indeed, in our method the only parameter to chose is the size of the block analysis. Obviously, it is easier to have an intuitive idea about the size of the homogeneous block than to chose the adequate value for κ_s as in the other similar methods where one has to try many κ_s values before arriving to the optimum one. Note that only one trial is necessary when using our ANAD method, whereas many trials, at least five in these examples, when using the other cited methods. It is worth noticing here, that the optimum results, on the basis of NMSE and subjective visual criterion, when using the other compared methods are shown.

6- Conclusion

Through this study it is demonstrated that the proposed improvements of the non linear anisotropic diffusion method allow to better control the amount of smoothing effect. Three main objectives have been achieved. The first concerns the introduction of a new bell-behaved conduction function sharing some interesting properties. The second is the way to control the pin-hole effect. Indeed, it is shown that all the other nonlinear diffusion methods known at present may suffer from this degradation phenomenon, not signaled before. The proposed method gives a robust solution to this problem. The third improvement we have made concerns the strategy for estimating the edgeness threshold. By making some simple and plausible assumptions on the signal and the noise one can automatically adapt the κ_s parameter during the diffusion process. Thus, in contrast with the other methods, the edgeness threshold becomes signal dependant leading to a better controlled diffusion. In summary, we have proved that some of the common drawbacks of the other compared nonlinear filtering methods can be overcome.

Some opened questions will be considered in a near future. It essentially concerns the strategy to adopt for adapting the block size as the diffusion process evolves.

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